SOME ACOUSTIC CHARACTERISTICS OF THE TIN WHISTLE

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1 INTRODUCTION

The sustained excitation of a tuned resonator by shed vorticity in a separating shear layer[1]or the whistling produced by the impingement of thin fluid jets on an edge[2] have both been exploited by the makers of musical instruments from time immemorial. Familiar examples include pan-pipes, recorders, flutes, organ flue pipes[1-3], and so on. Over the centuries, the acquisition of the necessary knowledge and skill for their successful production must have been laboriously accomplished by much trial and error. A more physically explicit understanding of the basic controlling mechanisms began to emerge during the great upsurge in scientific observation and discovery from the mid-19th century, as this was also accompanied by the relevant developments in physics, acoustics and fluid mechanics. These mechanisms can take several forms, depending on subtle differences in local and overall geometric detail and its relation to the magnitude, direction and distribution of any flow that is generating sound. One such form includes many examples of reverberant systems, where separating shear layers[3,4] provide the conditions where this coupled flow acoustic behaviour may occur. It is well known[1-4] that whenever a flow leaves a downstream facing edge it separates, forming a thin shear layer or vortex sheet. Such sheets, which involve high transverse velocity gradients, are very unstable and rapidly develop waves[1-4]. This behaviour, for example, resembles the flapping of a flag in the wind. Rayleigh[1] studied the stability of such sheets showing that either a sinuous or a varicose wave motion may develop in such jets of air. Thus they may in time roll up to form an ordered train of vortices. The interaction of such flowing fluid sheets or structures with an edge provides the basic physical mechanism that was observed to produce edge tones. The mathematics describing the physical and flow processes concerned with the generation of such tones tends to be strongly non-linear. The observed behaviour[2] demonstrates that both the sound intensity and pitch of these tones or whistles exhibit staging, controlled by jet velocity, with its profile and the distance and relative position of the edge from the jet orifice. Musical tones are produced by the strongly coupled aeroacoustic interaction of an edge tone generator with its appropriately tuned resonator. Observations[3,4] have also demonstrated that the sets of mathematical expressions that describe the acoustic behaviour of resonators, normally tend to remain essentially linear.

The mouthpiece and generator of a flageolet (tin whistle), or of a recorder, consists of a narrow rectangular passage or flue that forms the air jet that flows across the mouth, a rectangular opening in the wall, of the attached resonator, to then impinge on the wedge at its far side. The ratio of width to height of the passage, or its aspect ratio, typically exceeds five, while that of the mouth opening usually lies between one and two, although may exceed two with some organ pipes. The flageolet resonator continues as a uniform, or slightly tapering straight tube extending from the mouth to its open end, that is drilled with a set of six tone-holes. Other instruments, such as the ocarina, have a similar generator, but the resonator is egg shaped. With flue organ pipes, the flue may be simply a slit shaped orifice in a baffle between the pipe and its conical foot, resulting in a very short passage. The drilling of sets of holes in the wall of wind instrument resonators to provide the sequence of notes creating a musical scale is also a practice lost in the mists of time !. The appropriate position and size of the individual holes must have been established empirically by each skilled maker of the many former and existing types of wind instruments. It happens that the acoustic behaviour of a single hole in a pipe wall differs somewhat from that of a row of holes, so the development of each class of instrument must have involved much experimental endeavour. It also turns out that though the magnitude of the mean airflow velocity along the bore remains small, its presence does influence the acoustic behaviour to some extent, so may not always be neglected. It has been common practice to model the acoustic characteristics of flutes, recorders, pipes and whistles by adopting an electro-acoustic analogy[5], at least so far as this concerns the influence of opening or covering finger holes on the basic tuning or frequency of the resulting tones. Such modelling can remain realistic, if this is concerned solely with the linear response of the resonator while omitting flow. But changes of pitch, or tuning, with flow at the mouth, or with blowing pressure, are a widely recognised feature of wind instruments generally and may require non-linear or empirical features in any models that represent such behaviour adequately. Therefore, in what follows here, a different approach has been adopted that is based on predictive fluid dynamic modelling[4,6], that has been widely validated experimentally.

2 ACOUSTIC MODEL

The flow of both mean and fluctuating mass, energy and momentum is conserved within an appropriately positioned control volume situated at each area or other discontinuity throughout the instrument. Since the bore of the resonator tube is a small fraction of the acoustic wavelength that is of interest, it is assumed that the fluid motion will remain effectively one dimensional. The spectral characteristics of the fluctuating acoustic pressure p and particle velocity u that is associated with the wave motion can then be described in terms of the incident and reflected component wave complex amplitudes, p^+ and p^-, respectively. With plane waves one finds

$$p = p^+ + p^-, \rho cu = p^+ - p^-, \qquad (1, 2)$$

where ρ is the density, c the speed of sound and ρc the characteristic impedance of the air in the pipe. Wave reflections will occur at each area or other discontinuity, with the local reflection coefficient r, defined by the complex ratio $r = p^- / p^+$. Alternatively, at an open end of a pipe, for example, it is equal to the complex ratio that is defined by $r_e = -exp(-2i\psi)$, where $i = (-1)^{0.5}$. The associated inertial and similar influences on the wave transfer are taken into account by the inclusion of appropriate end corrections[6]. These then determine the axial position of any control volume or plane where wave transfer occurs. The ratio $p/\rho cu$ then defines the local dimensionless impedance ς. With one dimensional, or plane wave motion this can also expressed by

$$\varsigma = (1 + r) / (1 - r), \text{ or, } \varsigma_e = \tanh\psi = (\theta - i\chi) \qquad (3, 4a, 4b)$$

where ς_e represents the radiation impedance ratio of the open termination The development of an acoustic model for the flageolet will concentrate first on one describing the tuning of the simple pipe, or resonator, with all tone-holes covered and then go on to consider next the influence on tuning of a single tone-hole, followed by combinations of two or more such holes. Finally, the model should include any influence on tuning arising from the jet flow and associated edge tone production at the generator.

It is common experience that the measured frequency of the first sounding note, or fundamental tone of the flageolet, when all holes are covered, always has a half wave length distinctly longer than the overall length of the resonator. This is also true for many other wind instruments, such as the recorder, or the open organ flue pipe. This discrepancy can be resolved by establishing the actual effective acoustic lengths corresponding to the harmonic sequence of acoustic resonances, by including appropriate end corrections[1,6]. These will each represent the processes concerned with wave reflection from and transfer across area and other discontinuities, including sound radiation from the open end of the pipe, from any of the open tone-holes, and from the generator. A practical understanding of such appropriate geometric adjustments as end corrections and the like, that were necessary to achieve the desired musical scales, must also have been developed by the successful makers of wind or brass instruments, and pipe organs over ages past. It still remains common practice to include some provision for fine tuning by an individual performer on many, if not all, wind or brass instruments. Also, in many such, the tube wall at the open end is thickened, fitted with a bulge, rim or a flare, both for improving coupling with the surrounding atmosphere and modifying tone colour as well as establishing its characteristic individual appearance. However, the flageolet normally has a simple open end to the thin walled metal tube forming the resonator. For a thin walled or sharp ended pipe, a close approximation to the end correction l_{ec} is given by

$$\frac{l_{ec}}{a} = \frac{0.6133 + 0.027(ka)^2}{1 + 0.19(ka)^2}, \tag{5}$$

where *a,* is the tube radius, *k* the wave number *ω*/c, with *ω* the radian frequency and *ka* is the corresponding Helmholtz number. The end correction *l* introduces a phase shift that is equal to *exp(-2ikl)* which is also the phase of the corresponding complex reflection coefficient. This can be derived from the radiation impedance by rearranging equation 3.

Careful observations[8], with all tone-holes covered, revealed that this correction for the open end did not account for more than roughly one seventh part of the difference between the acoustic and physical length of the flageolet, a fact that was in agreement with similar observations with organ pipes. Thus a major part of the difference was presumably associated with edge tone generation. It is convenient to defer any further study of this now, but consider next a model describing wave transfer and sound emission at one or more tone-holes. To illustrate the basic behaviour, a close approximation to the observed behaviour results if one assumes that both the mean and the acoustic motion along the bore remain one dimensional, except in the vicinity of each hole. For a hole of area S_h in a pipe with local bore area S_p, conservation of fluctuating mass and energy associated with the acoustic motion at the hole is expressed respectively by

$$S_p u_2 = S_p u_1 + S_h u_h, \quad p_2 = p_1 = p_h, \tag{6,7}$$

where u is the local space averaged acoustic particle velocity with p the corresponding pressure, and the subscripts *h,* 1and 2 refer respectively to the hole *h* and the lengths of pipe either side of it. In terms of the corresponding incident p^+ and reflected p^- spectral component wave amplitudes, conservation of fluctuating mass and energy in the neighbourhood of the hole are expressed respectively[6] by

$$(1+M)p_2^+ - (1-M)p_2^- = (1+M)p_1^+ - (1-M)p_1^- + (S_h / S_p\zeta)(p_1^+ + p_1^-) \tag{8a}$$

$$(1+M)p_2^+ + (1-M)p_2^- = (1+M)p_1^+ + (1-M)p_1^-, \tag{8b}$$

where the Mach number $M = u_o/c_o$, with c_o the local speed of sound, ρ_o the corresponding density, u_o the mean flow velocity and $\varsigma = p_h/\rho_o c_o u_h$, while the value of the characteristic impedance $\rho_o c_o$ is assumed to remain constant. The wave transfer at the hole is then described by

$$p_2^+ = p_1^+ + S_h(p_1^+ + p_1^-)/(2S_p\zeta(M+1)), \tag{9a}$$

$$p_2^- = p_1^- - S_h(p_1^+ + p_1^-)/(2S_p\zeta(M-1)), \tag{9b}$$

The presence of the hole adds its end correction l_h to the resonator tuning length. The equivalent phase addition, *exp(-2ikl$_h$),* introduced by the presence of the hole, is again equal to the phase of the complex factor expressing the value of the reflection coefficient p_2^- / p_2^+.

Systematic studies[9] have provided appropriate quasi analytic expressions for the evaluation of the impedance ratio ς of a tone or tuning hole, or row of such holes, in the wall of a tube subject to grazing flow. They show that this flow may be neglected if M < 0.025. However, the value of ς is also strongly influenced[9] by the details of the geometry in the vicinity of the hole. The fluid inside the orifice can be modelled simply as a small piston of air. Its impedance is evaluated by quantifying the viscous, thermal and mass effects in the oscillating fluid. The influence of such holes on tuning also depends to some extent[10] on the precise nature of the source of excitation and also on the resonator geometry. The studies[9] involved comparisons of an extensive sequence of models and showed that the quasi analytic model described below gave a general description of hole impedance that remained in good agreement with observations. In accordance with observed behaviour, such impedance is normally modelled in two regimes. When, in the absence of grazing flow, the fluctuating velocity amplitude in the holes is less than about two metres a second, observations show[9] that the corresponding impedance exhibits linear behaviour, while at higher velocities this becomes nonlinear. This limit corresponds to a fluctuating pressure amplitude, or SPL of around 110 dB at 500 Hz, but exceeding 140 dB above 1 kHz. Also, experimental evidence[9] suggests that the limiting velocity just stated increases significantly with grazing flow, Since the limit depends on the velocity through the holes, the porosity σ, defined as the total area of the holes per unit surface area, must also be a factor, which therefore has been included. For the range of frequencies and hole diameters of interest, one finds that $|k_s r_o| > 10$, with $k_s = (-i\omega/v)^{0.5}$, the viscous wave number, where v is the kinematic viscosity and r_o is the hole radius . With M < 0.025, that is also relevant to the present case, the impedance ς can be expressed for the linear regime by

$$\zeta = \frac{l'}{c\phi}\sqrt{8\nu\omega} + \frac{1}{8c^2}(\omega\phi)^2 + i\frac{\omega}{c}\left(l'' + \frac{l'}{\phi}\sqrt{\frac{8\nu}{\omega}}\right),\ \sigma, \tag{10}$$

where $l' = l + \varphi$, $l'' = l + (8/3\pi)\varphi$, with l, the hole length or wall thickness and φ, the hole diameter. With a single tone-hole σ is equal to S_h / wp_{er} , where w is the hole axial width and p_{er} the local tube perimeter. With a row of closely spaced holes with differing diameters, the porosity σ, has been taken as the total area of the holes divided by the corresponding surface area, at a plane along the axis at the point of action of their moment of area. Widely spaced holes, where any mutual interaction is sufficiently small, may be assumed to act independently. Finally, the influence of the generator on the frequency of the tone generated can also be represented to a first approximation by a similar end correction, related to the radiation impedance of its rectangular opening, with effective diameter equal to its hydraulic diameter. The current studies reported here are restricted to the influence of flow and geometry on analytic predictive models describing the tuning, omitting at present any consideration being given to predicting the intensity of the radiated sound.

3 EXPERIMENTS

The measurements were all made on a tin whistle crafted by the Irish Company *Clare Whistle,* with a fundamental tuned to D_5. The body was formed from a brass tube 259 mm long inserted 24.5 mm into the plastic mouthpiece that was 59 mm long. The tube was not perfectly round, but on average it was just under 12.2 mm bore with an outer diameter just over 12.8 mm, giving a wall thickness of just over 0.3 mm. The tone-holes were also not quite round. Starting from the open end, their average hole diameters were respectively 6.61, 6.60, 4.97, 5.45, 6.03, 4.94 mm. The 1st hole was drilled 37.7 mm from the open end, with the centres of the remaining 5 spaced respectively 25.0, 18.5, 23.0, 23.1, 19.7, mm along the tube. The centre of the generator's rectangular opening was a further 120 mm from the centre of the 6th tone-hole, giving a total resonator length of 267.3 mm. The rectangular flue channel in the mouthpiece was 8 mm wide tapering from 2 to 1.48 mm deep and 24.55 mm long. This produced a jet of air flowing across the 5.3 mm long 8 mm wide mouth until it impinged on the 8 mm wide wedge. Its lower surface was a flat plane moulded so it formed a chord in the bore that lay just below the upper surface of the flue and parallel to the axis of the resonator tube. The resulting local sectional area of the resonator was reduced from that of the brass tube by the area of the segment bounded by the 8 mm chord, while the diameter of the corresponding bore was also slightly reduced, resulting in a local wall thickness of 2.4 mm at the rectangular opening. The upper surface of the wedge was 11 mm long and lay at an angle of about 25 degrees to the axis. Finally, the bore continued for 8 mm under the flue channel beyond its end, with a rapid but smooth reduction in its section area to zero. The whistle tests were performed while it was played in an anechoic room, using a regulated and metered air supply. The resulting sounds were monitored with a calibrated Knowles type CA-8374 electret microphone flush mounted over a 1.7 mm port drilled in the brass tube wall 25 mm towards the generator from tone-hole No. 6. Each pressure record was acquired on a National Instruments PXI-4472 system for 10 seconds at 10,000 samples per second with a 16 bit resolution and then processed to produce spectral records at a 2 Hz resolution over 5 kHz. The air temperature remained at 20 degrees Celsius throughout the tests.

3.1 Overblown response of a simple flue pipe

A first sequence of tests was made with all holes covered, to quantify the spectral response to a staged increase or decrease of flow rate in the flue and then to estimate the corresponding end correction at the generator. Figure 1 illustrates the resulting systematic steps in the frequency response to blowing volume flow rate, characteristic of edge tones and recognised[2,3] as staging. Of note is the significant hysteresis in the response to rising or falling flow rate, with the frequency on average proportional to flow rate, but having a much slower rate of change along each step.

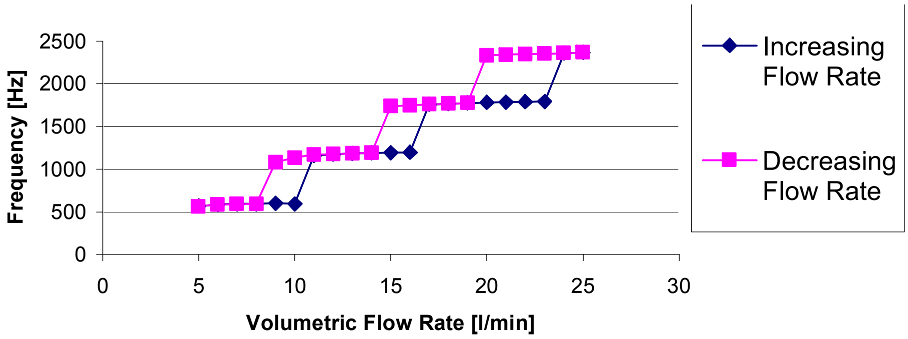
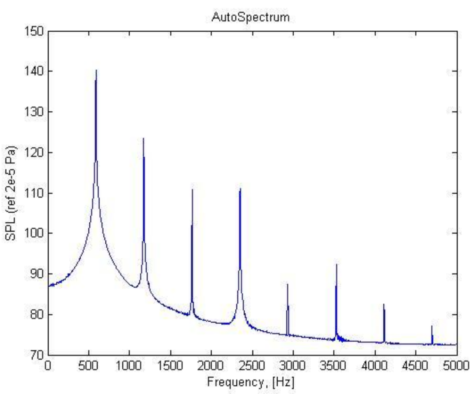
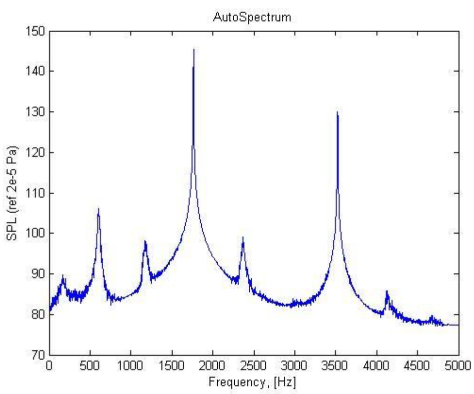


Figure1. Response of Clare Whistle D to blowing flow rate

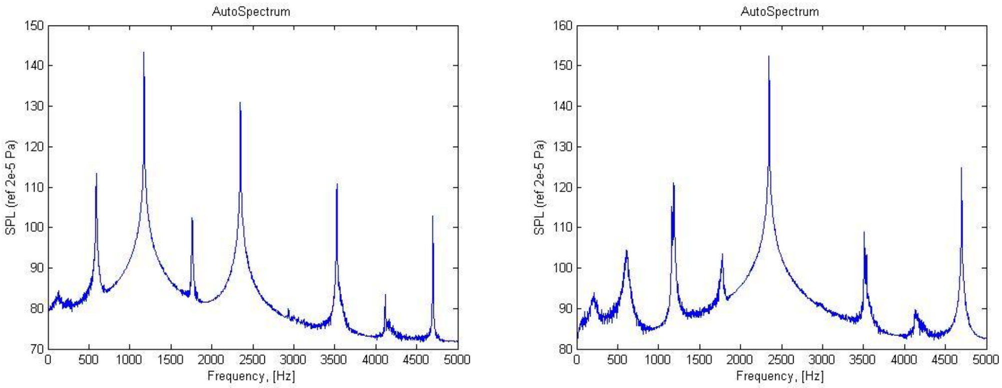
The corresponding measured frequency spectra corresponding to the fundamental tone, with the first three members of the over blown harmonic sequence, are presented in Figure 2. One notes that a full sequence of 8 components with steadily falling amplitudes is present in the D_5 spectrum,



1st D_5 587 Hz, 6.1 l/min



3rd A_5 1761 Hz, 17.9 l/min



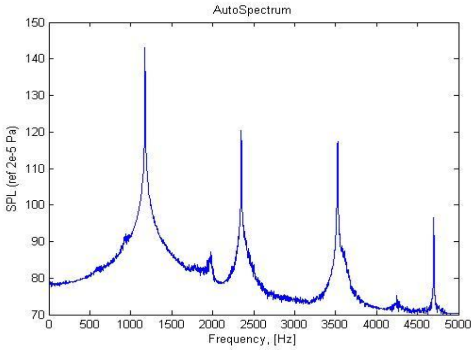
2nd D_6 1174 Hz, 12.0 l/min   4th D_7 2349 Hz, 23.8 l/min

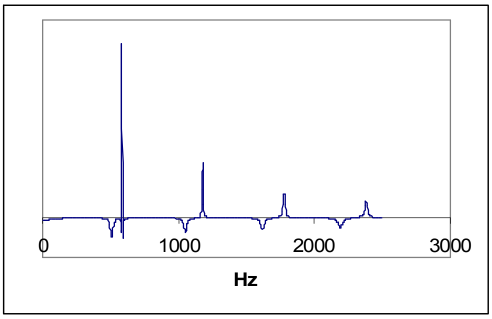
Figure 2. Overblown harmonic series of Clare Whistle D

while components 2,4,6 dominate D_6, 3,6 do so for A_6, and 4,8 the D_7 spectrum. Also, the corresponding flow volumes were almost exact multiples of the harmonic orders. This data can next be analysed to estimate end corrections at the generator, corresponding to each member of the harmonic sequence in figure 2. This involves a calculation of the end correction at the open end with Equation 5, then adding this to the measured resonator length and subtracting the result from 292.2, the product of the harmonic number with the acoustic half wave length for the corresponding tone. The value of the four corrections calculated for the open end reduces in steps of 0.02 from 3.74 to 3.68 mm, with a similar small rise in the resulting estimated value at the generator, with an average of 21.5 mm for the generator. One notes too that its value closely approximates the wavelength of the flapping instability in the shear layer, estimated as the ratio of average shear layer velocity to the frequency of the corresponding tone. In making this estimate, appropriate allowance was included to take account of boundary layer development in the flue. This wave would have a maximum transverse displacement at its quarter and its three-quarter wavelength and so on. Finally, one notes that the length of the mouth, or the free length of jet before it strikes the wedge, is also a close approximation to that of the quarter wave, which strongly suggests the probability that they were similarly related, at least as far as this instrument is concerned.

3.2 Observations with one or more tone holes open

The first octave from D_5 to D_6 of the normal major scale is normally played on the flageolet by opening the holes in succession to reach $C^{\#}_5$, the leading note. This sounds when all six holes are uncovered. The octave D_6 can then be played by over blowing with all holes closed again, but a cleaner tone is obtained if holes 1-5 remain covered while 6 is left open while blowing at the same air flow rate, as can be seen by comparing the spectra of D_6 in Figure 3 with that in Figure 2. A set of measurements was made for this fingering, after the airflow rate had been set to produce the corresponding frequency according to the tempered rather than the natural value for a D major. One might be reminded that the successive intervals in this scale are tone, tone and semitone, resulting in a perfect fifth, followed by tone, tone, tone and semitone. Each semitone interval is represented by the frequency ratio corresponding to the factor $2^{1/12}$, rounded to the nearest integer value. The





Right, D_5, predicted resonances, flow rate 6.2 l/min., with all holes closed
Figure 3. Left, D_6 spectrum, flow rate 11.9 l/min., with only hole 6 open

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predicted resonances for D_5 were calculated using the models described in section 2, to represent the measured flow and geometry of the Clare D Whistle with all holes covered. The plot shows the relative rate of change of the phase of the calculated input impedance at the exit of the flue. One can easily show that such distinct spikes correspond to resonances of a lightly damped system.

The measured behaviour[8] of the flageolet includes the spectra and associated flow rate for each note of the first octave with normal fingering, adjusting the flow to match the response to achieve the standard frequency in each case. These were repeated with a fixed flow rate of 9.5 litres per minute, recording the actual frequency of the notes obtained, which differed in most cases from the standard values. Both sets of measurements were also repeated[8], with only a single tone hole uncovered. Predictions of the resonances with the standard fingering and flow rates were also made with the model. These results, with those of all the measurements, are set out in Table 1.

	Standard Fingering					One Hole Open		
	Freq.	Holes open	Flow rate	f_{pred}	$f_{9.5}$	Hole open	Flow rate	$f_{9.5}$
	Hz		l/min	Hz	Hz		l/min	Hz
D_5	587	0	6.2	588	598			
E_5	659	1	6.9	659	612	1	6.9	672
$F_5^{\#}$	740	1-2	9.0	739	741	2	9.5	738
G_5	784	1-3	8.2	784	791	3	10.3	778
A_5	880	1-4	10.0	880	870	4	12.4	853
B_5	988	1-5	9.55	988	967	5	13	953
$C_5^{\#}$	1109	1-6	11.3	1110	1042			
D_6	1174	6	11.9	1174	1087	6	12	1087
D_6	1174	0	12	1174	-			

Table 1. Flow rates and frequencies observed while a Clare D Tin Whistle is played

The first six columns of Table 1 summarise the results obtained with the standard fingering. Columns 1and 2 list the names and corresponding frequencies of each tone for the major scale beginning with D_5. The tone holes that are open when playing each note of the scale are specified in column 3, with the corresponding air flow rates in column 4. One should take note of the general trend of increasing flow with rise in pitch. An experienced player routinely exploits this fact to adjust or correct the intonation or pitch when playing each note of a musical phrase. Column 5 sets out each predicted frequency calculated with the models described in section 2, for the measured geometry of the Clare D Whistle and the conditions set out in columns 3 and 4. Column 6 then sets out the frequencies measured with a constant rate of volume flow set at 9.5 litres per minute. This illustrates the relative sensitivity of the sounding frequency to either an enhanced or a reduced flow rate that is also seen in Figure 1. The remaining three columns give the corresponding results when only one hole is uncovered at a time. Column 8 records the flow rate required to play tones at the standard pitch with this restricted fingering. Conditions for E_5 and D_6 are obviously the same, while playing the remaining 4 tones at the required pitch requires a further increase in the air flow above the values listed in column 4. Finally, the corresponding set of 4 observations in column 9, that were also made with a constant flow rate of 9.5 litres per minute, also reveal trends in rising or falling pitch with flow rates that are similar, but exceed those recorded in column 6.

4 DISCUSSION

The measurements and modelling described here have successfully established some features of the coupled generator/resonator acoustic characteristics of the flageolet. This has mainly concerned the observation and prediction of its frequency characteristics, where the physical behaviour is effectively linear, while the source of the acoustic energy production is confined to the non-linear aeroacoustic processes associated with an edge tone mechanism at the mouth. These involve the production of a jet of air by the flue, flowing across the mouth to interact with the wedge, the resulting disturbances are then fed back to excite waves in the jet shear layer. The staging illustrated in Figure 1 combined with the spectra recorded in Figure 2 describes the extent to which the edge tone mechanism and the acoustic resonances are inherently related. This characteristic is one that is exploited by every performer with the instrument, to adjust the pitch of each note and extend the compass beyond the first octave. This requires a continuous adjustment to the flow rate accompanied by the appropriate fingering. These features are summarised in Table 1 for the octave D_5 to D_6, where the flow rate is seen to increase steadily with pitch. The results in column 5 illustrate how effectively the fluid dynamic model described in section 2 predicts the resonances associated with each flow rate and fingering. The remaining data in this table reinforces the demonstration of the strong coupling that exists between flow rate and pitch for each fingering. The fluid dynamic model also calculates the end corrections associated with the open end, the tone holes and the generator. Combined with the flow rate measurements it also provides further insight into the physical processes concerned in the aeroacoustic generation of tones by the instrument.

Mouth End Corrections					Shear Layer Wavelengths			
	Acoustic half wave length	Corrected resonator length	Mouth end correct'n	End correct'n for holes	Flow rate	Mean velocity of shear layer	Sound or shear wave frequency	Length of shear waves
	mm	mm	mm	mm	l/min	m/s	Hz	mm
D_5	292.5	270.7	21.8	-	6.2	12.9	587	22.0
E_5	260.6	239.5	21.1	6.5	6.9	14.4	659	21.8
$F_5^{\#}$	232.0	211.4	20.6	3.4	9.0	18.7	740	25.3
G_5	219.0	198.2	20.8	8.6	8.2	17.1	784	21.7
A_5	195.1	174.7	20.4	8.0	10.0	20.8	880	23.3
B_5	173.8	153.2	20.6	9.9	9.55	1.9	988	20.1
$C_5^{\#}$	154.8	134.2	20.6	10.5	11.3	23.5	1109	21.2
D_6	146.2	126.0	20.2	2.3	11.9	24.8	1174	21.1

Table 2. Calculation of end corrections and shear layer transverse wave lengths at the mouth

The end corrections at the open end remained close to 3.7 mm yielding values at the mouth for each of the first four harmonics shown in Figure 2, that were 21.46, 21.48, 21.50 and 21.52 mm respectively, with an average of 21.5 mm. The corresponding values when playing the scale D_5 toD_6 with the standard fingering, are summarised in Table 2. These were estimated from calculations that included the appropriate end corrections for the tone holes and that for the open end. Here column 2 lists the acoustic half wavelengths of each tone, while the corrected physical lengths of the resonator excluding that for the generator mouth are set out in column 3. The difference between the two columns is then the end correction for the mouth that is listed in column 4. The corresponding end corrections calculated for each single or rows of holes follows in column 5. When estimating their values, due allowance was made for any acoustic interference between them, or in the case of the first hole in particular, with the near field of the open end nearby. One notes too that when playing D_6, the tone hole effectively bisects the resonator tube.

The next four columns in Table 2 summarise the calculations of wave development in the shear layer at the mouth. Column 6 repeats the flow rate data from Table 1. A flow rate of 12 litres a min. corresponds to an average velocity of 17.4 m/s in the flue, which has an exit area of 11.5 cm^2, if one neglects boundary layer growth. The corresponding hydraulic diameter of the flue is 2.48 mm, with a Reynolds number of 3000. Although this is seemingly a borderline flow, if one takes due account of the long approach tube from the flow meter, it seems most likely that the flow in the flue would be turbulent, which is also a more conservative assumption resulting in thinner boundary layers than assuming it remains laminar. For such channel flows at such modest Reynolds numbers, there is a universal velocity profile[11], which in the present case yields a ratio of boundary layer displacement thickness δ^* to flue semi depth h of δ^*/h = O.146. This reduces the net effective cross section area of the flue to 8 mm^2 with a corresponding increase in mean flow velocity to 25 m/s and with Reynolds number now increasing to above 4000. The resulting corrected mean velocity estimates for the flow in the flue are listed in column 7. Assuming a roughly V shaped initial velocity profile, the convection velocity of the two shear layers in the issuing jet will also lie close to that of the mean velocity in the flue. The corresponding length of any instability waves developing in the shear layer will then be equal to the ratio of convection velocity to frequency and these results are listed in column 9. With two exceptions at $F^{\#}_5$ and A_5, the values are strikingly similar to those listed in column 4. The two anomalies might suggest that their measured flow rates were slightly in error on the high side, a possibility suggested by the corresponding departures from a steadily rising progression implied by the rest of the values in column 7. Finally, if further investigation reveals a similar consistent equality between acoustic end correction at the mouth and shear layer wavelength exists with other edge tone generator/resonator geometries, this fact would represent another useful contribution to our understanding of the associated aeroacoustic processes concerned in tone generation. Similarly, whether the simple relation between mouth length and wavelength that clearly existed at each tone generated by this Clare tin whistle has any general significance, also remains to be established.

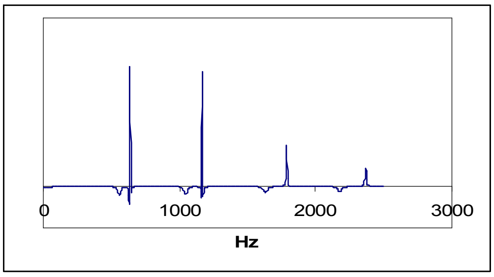

Figure 4. Predicted resonant frequencies with the standard fingering for D_6, flow rate 12 l/min

The results summarised in Table 1 record that the playing of the note D_6 requires twice the flow rate needed for D_5 with either the standard fingering, or when overblown with all the tone holes covered. The two resonances predicted for D_6 plotted in Figure 4, that occurred at 635 and 1174 Hz respectfully, had almost equal rates of phase change. This differs from the predictions for D_5 at a flow rate of 6.2 l/min in Figure 3 shown as an harmonic sequence from 587 Hz, where the decay in magnitude of the spikes at successive tones follows the same trend as that for the corresponding spectral amplitudes of D_5 seen in Figure 2. When these results are compared with the relevant observations in Table 2, it is clear that the most strongly excited tones in the resonator were always the ones preferentially coupled to the shear wave motion existing in the mouth of the edge tone generator, that was controlled by adjusting the appropriate rate of flow. This illustrates the dominance of such flow acoustic coupling that exists between shear waves present there and the dynamic response of the resonator in determining the frequency of acoustic waves that are most strongly excited in the resonator. Similarly, it demonstrates that the convection velocity of the shear waves is one essential factor that determines the actual pitch of the tones sounded by the flageolet at one of its acoustic resonances. Presumably this is also true for other wind instruments excited by an edge tone generator.

The measurements were all made in the Doak Laboratory which provides an effectively anechoic environment over the range of frequencies covered in the measurements. Thus pressure records remained free from contamination by room reflections or any other extraneous sources, apart from the inevitable presence of some low level jet mixing noise. All these records were obtained with a flush wall mounted and calibrated microphone and reduced to sound pressure level spectra. Therefore, they quantify the spectral levels observed at this fixed point rather than those associated with the standing acoustic waves in the resonator. To estimate these one requires further data, such as that provided by simultaneous pressure records obtained with a pair of appropriately spaced flush wall mounted calibrated microphones. Alternatively, the standing wave patterns may be described with reference to the flageolet geometry, once the values of the corresponding end corrections have been quantified, as they have been here. The resulting description of the acoustic climate within the resonator can then be compared with the corresponding predictions obtained with the quasi-linear model described above. The close agreement found between the predictions and the observed acoustic characteristics of the resonator described here, with some aspects of the associated coupled behaviour of the generator, is encouraging evidence of the effectiveness of the fluid dynamic approach adopted for the modelling. However, a complete description of the acoustic behaviour of the flageolet requires the measurement of the sources of sound emission to the surroundings, with their directivity. While a full understanding of its physical behaviour extends to the quantitative modelling of the aeroacoustic production of sound at the generator, as well as its subsequent emission from generator, the tone holes and the open end. This represents the challenge inspiring current developments.

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