Drum Head Modal Vibrations

Abstract:
The report considers the vibration mode of a membrane for different vibration frequencies. The focus is determining and comparing vibration modes of a drum head in order to characterize the acoustical properties of various shaped drumheads. Drums have been used all over the world and have a characteristic sounds, which many people enjoy. The report should give the reader an understanding of velocity and pressure fields associated with a vibrating drum.

Theoretic description of vibrating drum

The vibration of a drum can be explained by looking at the mathematics. If we consider a circular membrane with no displacement at the boundary also call Dirichlet boundary conditions. The membrane motion can be expressed as

$$\nabla^2 X(r,\theta,t). - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} X(r,\theta,t) = X_0 \exp(i\omega t)\delta^2(r),$$

where X is the displacement of the membrane from its rest position, v is the velocity, ω is the angular frequency, r is the radius and θ is the angle to the x axis. The left hand side of the equation describes the perturbation of the drum, which is a point like perturbation positioned at $r = 0$ ($\delta(r)$ is a two-dimensional delta-function with dimension $1/r^2$). The perturbation is a point like displacement and it oscillate with frequency. If we look at the equation for r greater than 0, the equation for the displacement is given by:

$$\nabla^2 X(r,\theta,t). - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} X(r,\theta,t) = 0$$

where the operator ∇^2 for a two dimensional problem is given by

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}.$$

Inserting this result together we get

$$\left[\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right] X(r,\theta,t). - \frac{1}{v^2}\frac{\partial^2}{\partial t^2} X(r,\theta,t) = 0$$

Assuming the displacement is oscillating with the same frequency as the applied displacement to the drum meaning that $X(r,\theta,t) = X(r,\theta)\exp(i\omega t)$ we obtain a time independent equation for the membrane vibration

$$\left[\frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right] X(r,\theta). + \frac{\omega^2}{v^2} X(r,\theta) = 0$$

We multiply the equation by r^2 and make separation of variables assuming $X(r,\theta) = R(r)T(\theta)$. The equation above then becomes

$$\frac{1}{R(r)}\left[r\frac{\partial}{\partial r} + r^2\frac{\partial^2}{\partial r^2}\right] R(r) + \frac{1}{T(\theta)}\frac{\partial^2}{\partial \theta^2} T(\theta) + \frac{r^2\omega^2}{v^2} = 0.$$

In order to solve this equation we know that the θ dependent part should be equal to a constant. We can therefore separate the r- and θ-dependence into two ordinary differential equations.

$$\left[\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2}\right]R(r)=\left(\frac{m^2}{r^2}-k^2\right)R(r)$$
$$\frac{\partial^2}{\partial\theta^2}T(\theta)=-m^2T(\theta)$$

with $kv=\omega$. We can now solve the equation for $T(\theta)$ which has the following solutions.

$$T(\theta)=\alpha_m\cos(m\theta)+\beta_m\sin(m\theta)\quad m=0,\pm1,\pm2,\ldots.$$

α_m and β_m are arbitrary constants which has to satisfy the following condition.

$$\sqrt{\alpha_m^2+\beta_m^2}=1.$$

Looking at the r-dependent equation

$$\left[\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial r^2}\right]R(r)-\left(k^2-\frac{m^2}{r^2}\right)R(r)=0$$

This equation is called Bessel's equation after a famous Friedrich Wilhelm Bessel a German mathematics. The solution is

$$R(r)=A_mJ_m(kr)+B_mY_m(kr)$$

where $J_m(kr)$ and $Y_m(kr)$ are Bessel's functions. The Bessel's functions are plotted in figure 1.

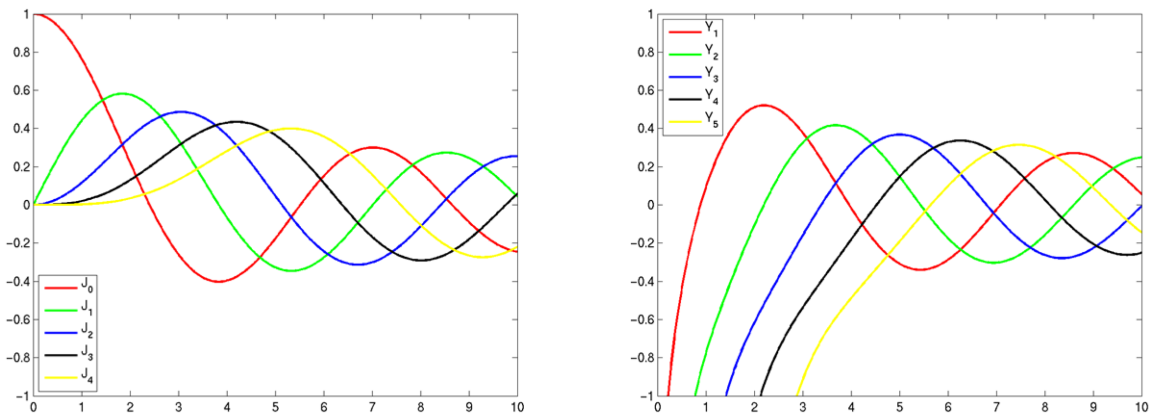

Figure 1: Bessel function's of 1st and 2nd kind. To the left the five 1st Bessel's function of first kind is shown whereas the right figure shows the first five Bessel's function of the 2nd kind. Note that Y_m diverges as r approach zero.

If we look at the two kinds of Bessel's functions Y_m cannot be a solution due to the fact that it diverges when r approaches 0. Therefore $B_m= 0$. If we look at J_m it has to vanish on the boundary of the membrane meaning that $J_m(kD)= 0$, where D is the radius of the drum. As we see on the plots in figure 1 there is more than one point where J_m is equal zero. The radius of the membrane is constant which means that there exist an infinite number of different k-vectors which satisfy the boundary condition. We can number those $K_{m,n}$ where n is the number of the nth zero of J_m. Using these considerations we obtain the solution for $X(r,\theta, t)$.

$$X(r,\theta,t)=A_{m,n}J_m(k_{m,n}r)[\alpha_m\cos(m\theta)+\beta_m\sin(m\theta)]\exp(i\omega_{m,n}t)$$

where $A_{m,n}$ is the arbitrary amplitude which is determined by the amplitude of the function generator. Note if the excitation is done at $r= 0$ we only exited the modal vibration which does not have a note at this point. It means the applied function generator as we use in this derivation has the solution

$$X(r,\theta,t) = A_{0,n} J_0 (k_{0,n} r) \exp(i\omega_{0,n} t).$$

In order to excite all the other modes one has to move the position at which the applied function generator acts to $r=d$. Note that the all J_0 are non degenerate where as all other modes is 2 times degenerated, due to the fact that α_m and β_m are independent of one another. In the experiments done in this report we only excited one vibration mode. This means that all other mode vibrating with other frequencies is largely suppressed and can be assume to be zero. In order to find the frequency, $\nu_{n,m}$, we used the

$$\nu_{m,n} = \frac{\omega_{m,n}}{2\pi} = \frac{v k_{m,n}}{2\pi}$$

This is the frequency one has to apply to the drum in order to get it vibrating in only one of all the resonant modes. Now we can calculate how the near field velocity field and pressure field looks just above the drum head. If we assume that the average air molecule above the drum stands still relative to the drum head we can associate the found displacement of the drumhead to be equal to the air molecules motion just above and below the drum. The displacement of the air molecules *s* is the given by

$$s(r,\theta,t) = A_{m,n} J_m (k_{m,n} r)[\alpha_m \cos(m\theta) + \beta_m \sin(m\theta)]\exp(i\omega_{m,n} t)$$

using Newton law's we can relate the displacement *s* to the velocity of the molecules *u*.

$$u(r,\theta,t) = \frac{\partial}{\partial t} s(r,\theta,t)\hat{z} = i\omega_{m,n} A_{m,n} J_m (k_{m,n} r)[\alpha_m \cos(m\theta) + \beta_m \sin(m\theta)]\exp(i\omega_{m,n} t)\hat{z}$$

where $\omega_{m,n}$ $A_{m,n}$ $=U_{m,n}$ that is the amplitude of the average velocity of the air molecules. The velocity is 90° out of phase with the displacement. We are only looking at the outgoing velocity field. We can now use the Euler equation to calculate the pressure associated with the velocity u.

$$\frac{\partial}{\partial t} u(r,\theta,t) = -\frac{1}{\rho_0} \nabla P(r,\theta,z,t) = -\frac{1}{\rho_0} \frac{\partial}{\partial z} p(r,\theta,z,t)\hat{z}\ .$$

We have made use of the fact that the velocity field has no θ-dependence. The pressure is then given by

$$p(r,\theta,t) = -\rho_0 \int dz \frac{\partial}{\partial t} u(r,\theta,t) = c\rho_0 \omega_{m,n} A_{m,n} J_m (k_{m,n} r)[\alpha_m \cos(m\theta) + \beta_m \sin(m\theta)]\exp(i\omega_{m,n} t)$$

where we have assume that $\omega\Delta z$ is equal to the speed of sound in air denote *c*, not to confuse with the speed of light in electromagnetism. Note that this solution breaks down close to the edges of the drum because the velocity field does not only point in the *z*-direction. The velocity field also becomes *r*-dependent, which complicate the derivation. This means that the pressure oscillated in phase with the displacement of the membrane. Most of this derivation is taken for Errede lecture notes [1]

As mention earlier the frequency of the drum depends on the shape of the drums, but the different resonant frequencies is not a multiple of the lowest resonant mode. The ratio between different vibrations frequencies can be found in book by Rossing et al. [2] for an ideal membrane. The number found in the book is shown in Table 1.

Table 1: The frequency ratio between different vibrations mode normalized to the J_{01}. The numbers are taken from Rossing et al. [2]

Mode	J_{01}	J_{11}	J_{21}	J_{02}	J_{31}	J_{12}	J_{41}	J_{22}	J_{03}	J_{51}	J_{32}	J_{61}	J_{13}
Ratio	1	1.59	2.13	2.29	2.63	2.90	3.14	3.49	3.59	3.63	4.05	4.14	4.22

Explanation and visualization of isospectral drums

Having worked through the theoretical derivation of how one would expect the displacement-, velocity- and pressure field associated with a drum we continue with another interesting phenomenon associated with vibrating drum, according to C. Gordon et al [3]. the spectrum of two different objects with same area and circumference is identical. If one is interested to see the derivation of this result one can look in this article, but note in order to understand the proof one has to know a bit of functional analysis.

This means that we are unable to hear the shape of a drum if it has the same area and circumference. Using COMSOL Multiphysics one can simulate that two membranes of different shapes emit the same spectrum. The membranes studied in this report are originally described theoretical by Driscoll [4] and experimental verified by Sridhar et al. [5]. Driscoll showed that the membranes shown in figure 2 have the same spectra.

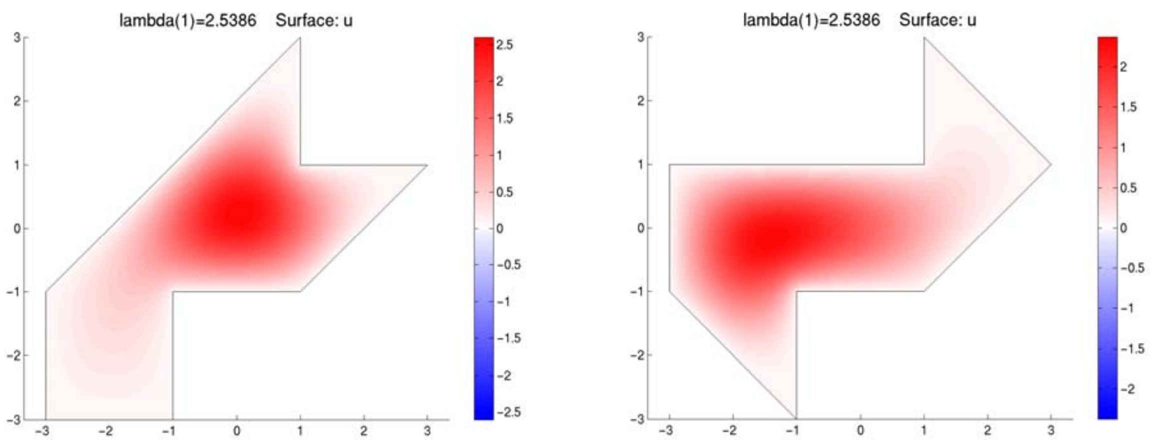

Figure 2: The velocity field of 2 different shaped drums. The red color means positive velocity whereas blue means negative velocity. The figure shows the 1st resonant frequency which is the same for both drums.

The simulation of the membranes is done by solving the differential equation inside the active area by means of finite element method so it satisfies the Dirichlet boundary condition. Figure 2 shows the velocity field for the 1st resonant frequency. As one see above the figure the frequency at which both drums vibrate is the same. The same is the case for the two next resonant frequencies shown in Figure 3 and Figure 4.

This result is bit surprising due to the fact that if one look at the first 3 resonant frequencies the velocity field for the two drum does not look similar at any point except that the number of anti-nodes is the same for the same frequency.

The result however is used in other fields than acoustics due to the fact that different shaped membranes with same circumference and area emits the same spectrum. It allows people to get a better understanding of different crystals might have similar structure.

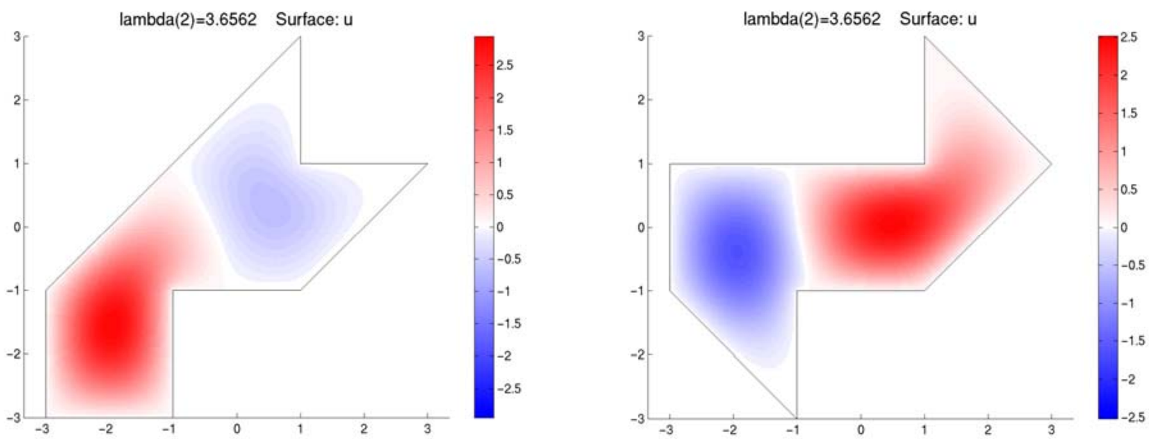

Figure 3: The velocity field is plotted for the 2nd resonant frequency.

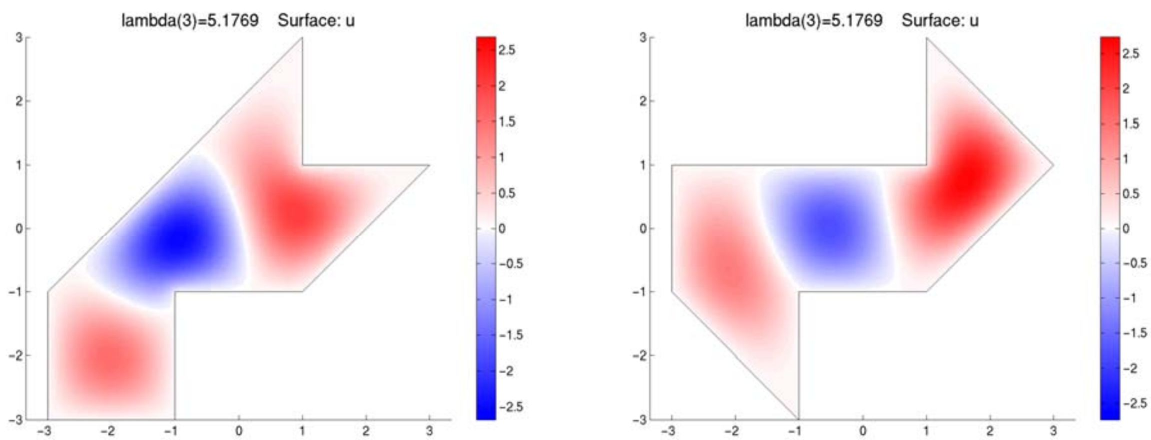

Figure 4: The velocity field is plotted for the 3rd resonant frequency.

Experimental setup

The experimental work is done on two drums, a doumbek and a Brazilian drum. The setup consists of an electromagnet that is placed a few mm below the membrane which is controlled by a function generator. The function generator is used to control the driving frequency as well as the power going to the electromagnet. On the way from the function generator to the electromagnet there is a small circuit which takes a flat current output as a function of frequency to a given power level. This allows the user to know that the current to the electromagnet is independent of frequency and thereby keeping the magnetic field induced constant. The width of constant current go from approximately from below 30 Hz to above 5000 Hz.

On the membrane of the drum there are placed two tiny magnets, one on each side. Placing these magnets on the membrane above the electromagnet the induced magnetic field will pull or push the magnets and thereby the drum back and forth with the frequency controlled by the function generator. The amplitude of membrane oscillation depends on the current applied to the electromagnet. Therefore by cranking up the power the amplitude increases which increases the pressure and particle velocity as shown in the in one of the previous sections.

Recording the pressure and particle velocity is done by using 4 microphones, 2 pressure microphones and 2 particle velocity microphones. The specification of the microphones is given in Errede lecture note [6]. Two microphones one of each kind is placed above or beneath the membrane, where the ones beneath are used to control the frequency of the drum and the two above are used to scan the entire surface of the membrane measuring 32x32 points separate by 1 cm. Each measurement of a drum takes some time due to the fact that between each measurement the microphones have to be moved to a new position. The signal from the microphones is send to a lock-in amplifier which amplifies the signal and generates the real and imaginary part of signal. Information about the lock-in amplifiers is given in Errede lecture note [7]
During this time period the resonant frequency can change due to changes in the temperature and humidity. Several measurements have shown that the setup is quite sensitive to static electricity and temperature changes. By having the two microphones beneath the membrane at the same position one can use these microphones to keep track of the resonance. At resonance the particle velocity and the pressure measured are 90° out of phase, which is also shown in the derivation in a previous section. By changing the frequency while measuring one can keep the drum vibrating at resonance but different vibration modes has different parity that one has to take into account when changing the frequency.
All this keeping track of resonance, moving microphones and storing data is done by a computer. Afterwards the data analysis is done in Matlab.

Investigation of a doumbek

The first measurement is done on the doumbek, which is an instrument from the Middle East and Egypt. The drum is so slim that we have troubles to get all the microphones and electromagnet all the way up to the membrane. The particle velocity microphone beneath the membrane is placed further away from the membrane but inside the drum. This could lead to some artifacts in the measurements.
The first thing to do is to find the resonant frequencies of the drum; this is done by making a frequency scan to see where the constructive and destructive interference between the drum and the function generator is. The resonance is where there is a local maximum interference between them. The particle velocity and pressure below the membrane is shown in Figure 5 as a function of frequency.

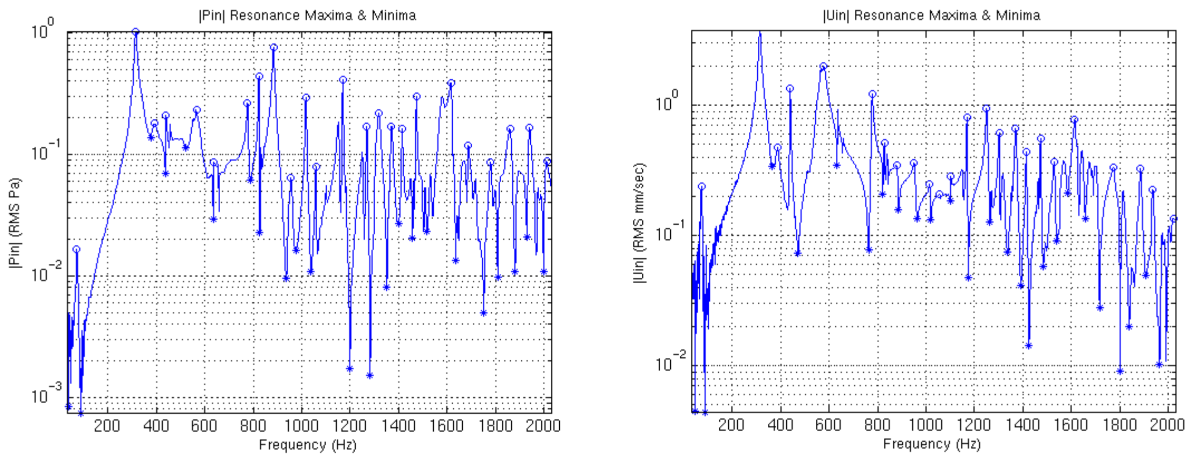

Figure 5: RMS values for the amplitude of the pressure (right figure) and particle velocity (left figure) inside the doumbek. The amplitude peaks are denoted by a circle and anti-peaks are denoted by a star. The measurement is done with 1 Hz step.

The amplitude of the pressure and velocity field determines where the resonances of different modes occur. All resonant frequencies are marked by a circle whereas crosses denote frequencies with anti-resonances occurring. The resonant frequencies are the one people most likely hear while playing the instrument because they are very easy to excite. The mode of the following 6 different frequencies has been measured: Circle number 2 at 313 Hz, number 4 at 440 Hz, number 5 at 565 Hz, number 7 at 787 Hz, number 8 at 845 Hz, number 9 at 962 Hz.
In Figure 6 the frequency and phase variation during each of the 1024 measurements. In order to get a good measurement one need the variation in frequency and phase as the case is in this measurement. The mode is generated by applying a voltage from the function generator V_{FG} = 3.5 V. The temperature was 19.9° C, the pressure 738 mmHg (1 atmosphere = 760 mmHg) and the relative humidity 31 %. These weather dependent variables seem to have an impact on the measurement.

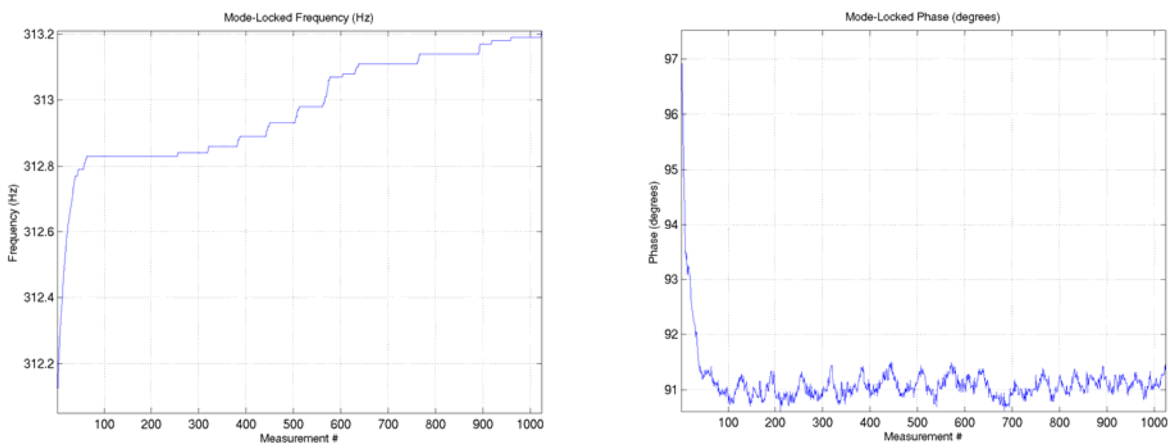

Figure 6: To the left is the frequency flow and to the right is the phase change of the resonant frequency while measuring. The resonance has a J_{01}-mode with positive parity.

Figure 7 shows the imaginary part of the pressure and the real part of the velocity of air molecules. The measured amplitudes are root mean squared (RMS) values in all the plots shown in this report. The plots show the J_{01}-mode which is the lowest resonance mode of a circular membrane. As shown the in the theory part the pressure is 90° out of phase with the velocity.

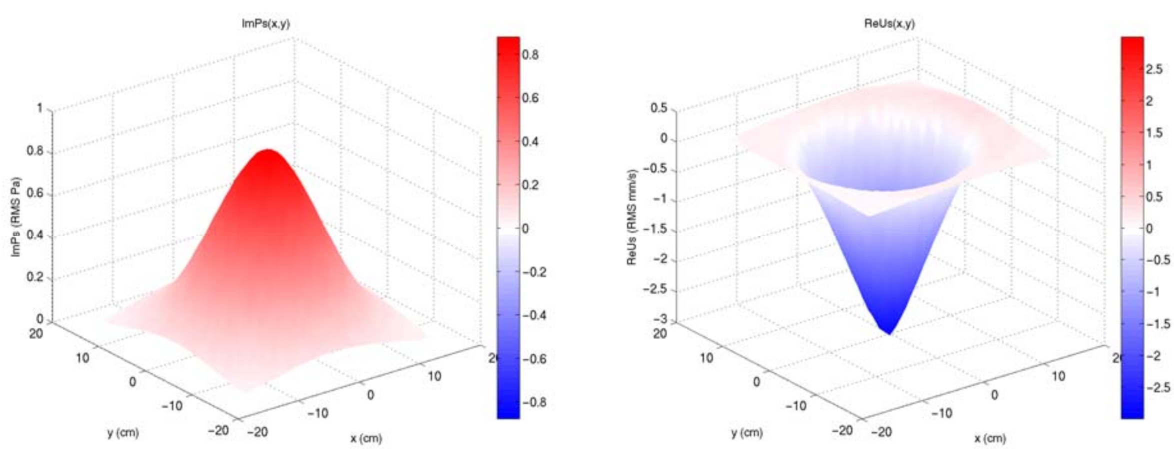

Figure 7: The RMS pressure (left) and velocity of air molecules (right) is shown for the J_{01}-mode of a doumbek. It is the imaginary part of the pressure and the real part of the velocity.

In Figure 8 the imaginary part and the velocity amplitude of air is showed. In the amplitude picture one can see where the boundary of the membrane is. It is where one has zero particle velocity. Further note that the measured velocity is the velocity perpendicular to the membrane. The velocity of air going in the plane of the membrane is not determined. This could be the reason for small derivation between the pressure and the velocity field due to the fact that the pressure microphone is unidirectional allowing us to get the total pressure.

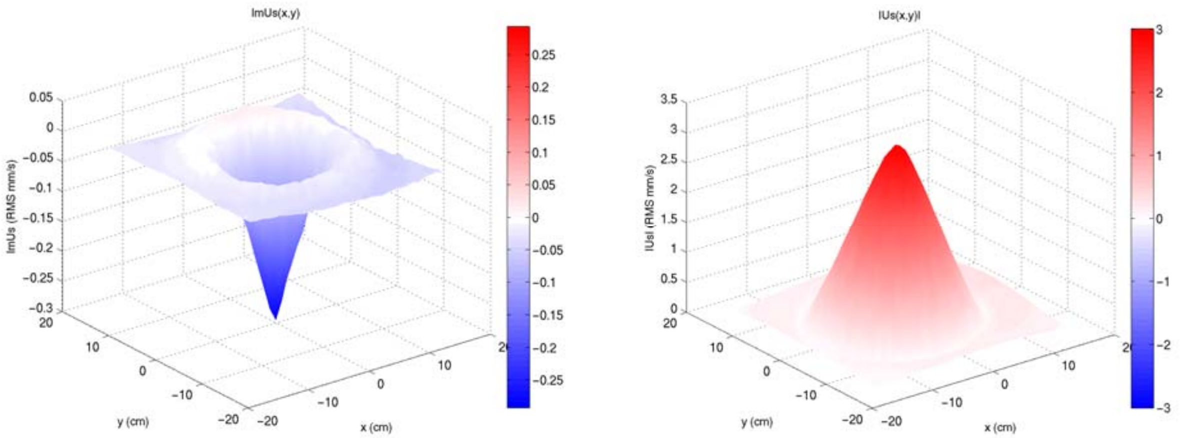

Figure 8: The imaginary part and the velocity amplitude of air above the membrane are showed. The maximal velocity of air is 3 RMS mm/s, which is in the center of the drum.

The theoretical derivation showed that the pressure was in phase with the displacement of the drum membrane and thereby 90° out of phase with the velocity shown in Figure 9. Therefore our derivation of how the displacement of a drum membrane generates a pressure and a velocity field is correct. There is a lot of other interesting quantity one could show how looks. In examples one could look upon the impedance and the intensity of the drum, but due to not putting too many pictures in to the report and the fact that all other quantities can be derived by the pictures given, the discussion is left out.

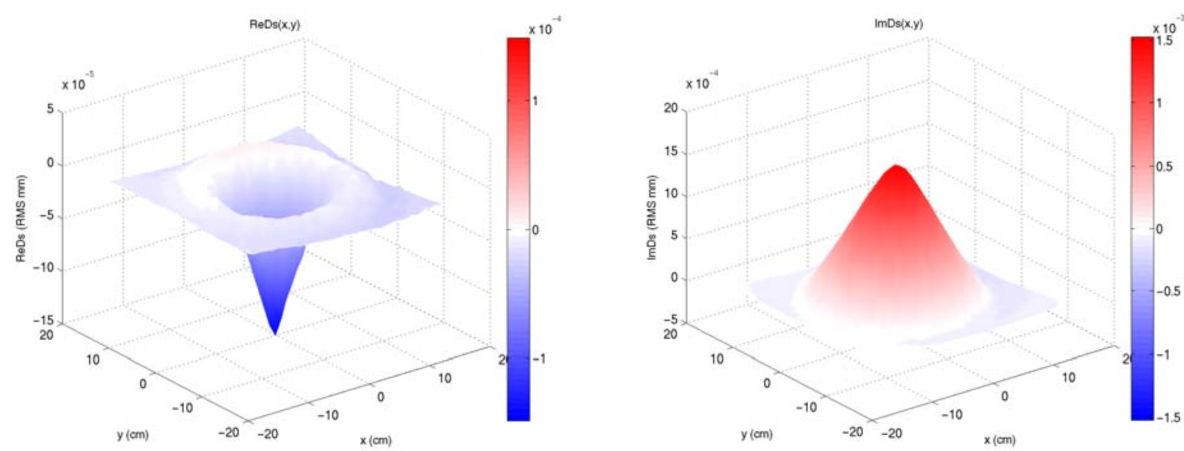

Figure 9: The real and imaginary part of the displacement of the drum membrane is shown. The Displacement field is 90° out of phase° out of phase with the air molecules velocity.

The next mode is the J_{11} shown in Figure 10 and Figure 11. Figure 10 shows the frequency and phase changes during the measurement. The applied power form the frequency generator is V_{FG} = 3.5 V.

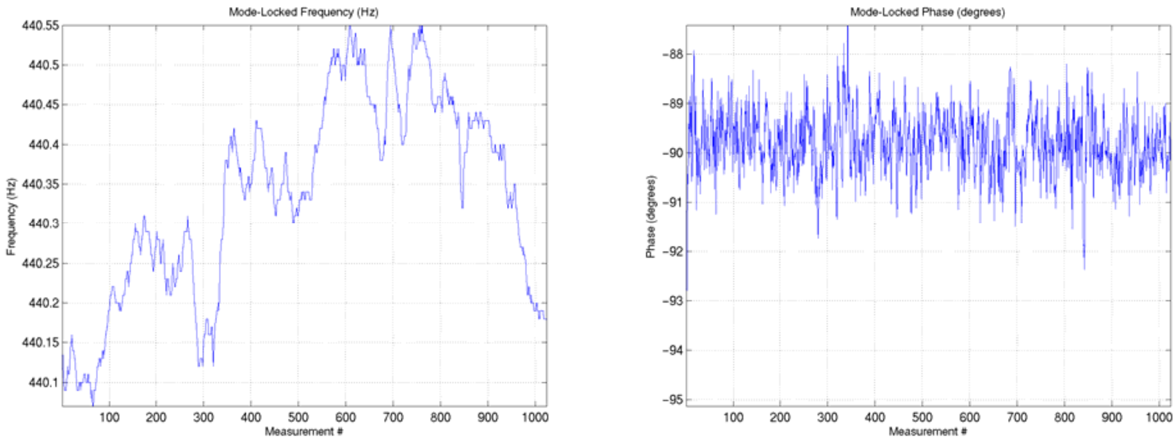

Figure 10: The frequency and phase changes while measuring on J_{11}. Temperature 17.3° C, pressure 745 mmHg and relative humidity 26 %. The J_{11}-mode has a negative parity.

The real part of the velocity field is a J_{11}-mode, whereas the imaginary part looks different. The Vibration mode is two times degenerated, meaning that if we rotated the image 90° in the (x,y)-plane we would obtain the other J_{11}. The average frequency is 440 Hz which means that the ratio between J_{11} and J_{01} is 1.41 which is more than the ratio of an ideal drum given in Table 1

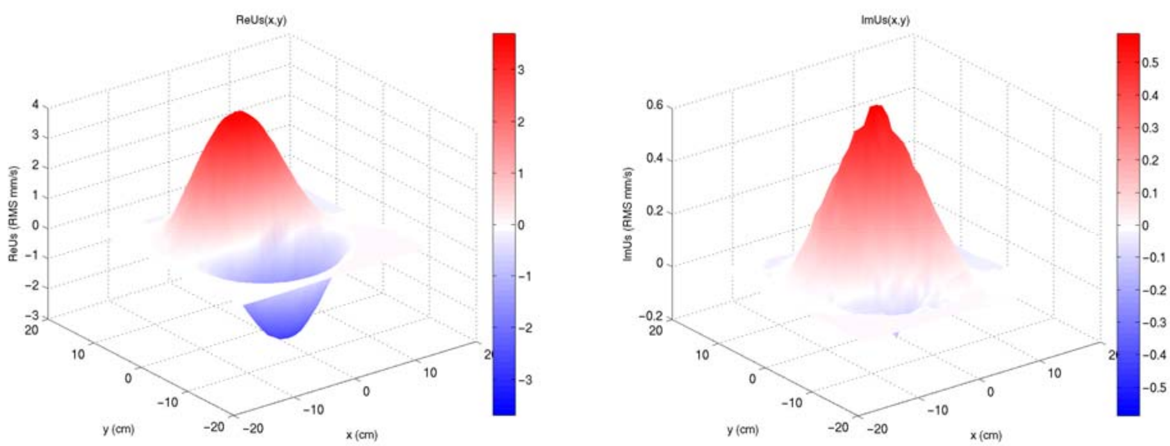

Figure 11: The real and imaginary part of velocity for a J_{11} of a doumbek.

The next mode measured it the J_{02} where the frequency and phase changes during measurement is shown in Figure 12. The frequency and phase variations while measuring is not as stable as for the lowest two modes and the frequency seems to drift while measuring which can be induced by changes in the climate. According to our ideal drum this frequency should be a J_{21}. This could be explained by a large black circle in the middle of the drum which acts as a damping source and the reason that the shape of the drum does not allow us to move the position at which the membrane is excited.

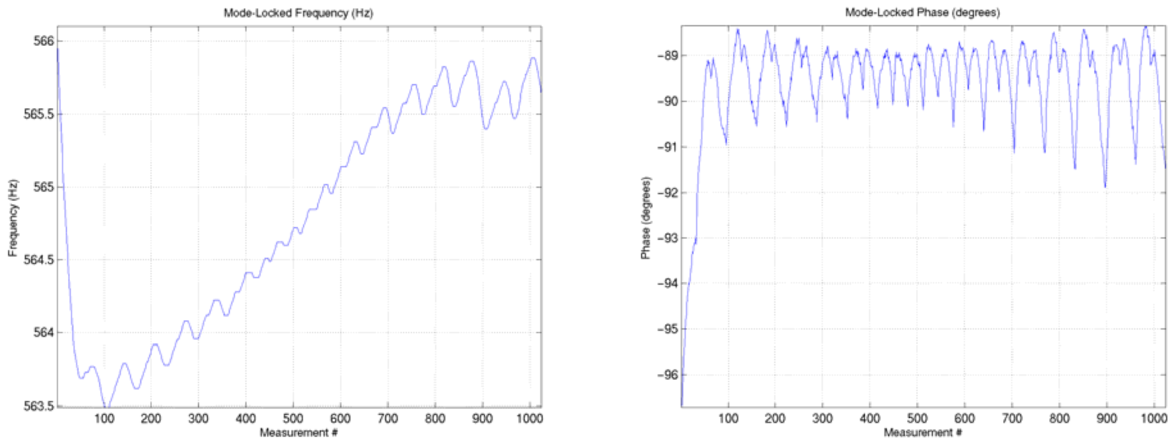

Figure 12: The frequency and phase of J_{02} are shown. The temperature is 17.3° C, the pressure is 745 mmHg and the relative humidity is 25 %. V_{FG} = 3.5 V. The J_{02}-mode has a negative parity.

Figure 13 show the real and imaginary part of the velocity field perpendicular to the drum. The real part of the drum one can see the negative peak in the center that is 90° out of phase with the positive ring peak surrounding it. The ratio between the frequency of J_{02} and J_{01} is 1.81 which is less than the ratio obtained by an ideal membrane.

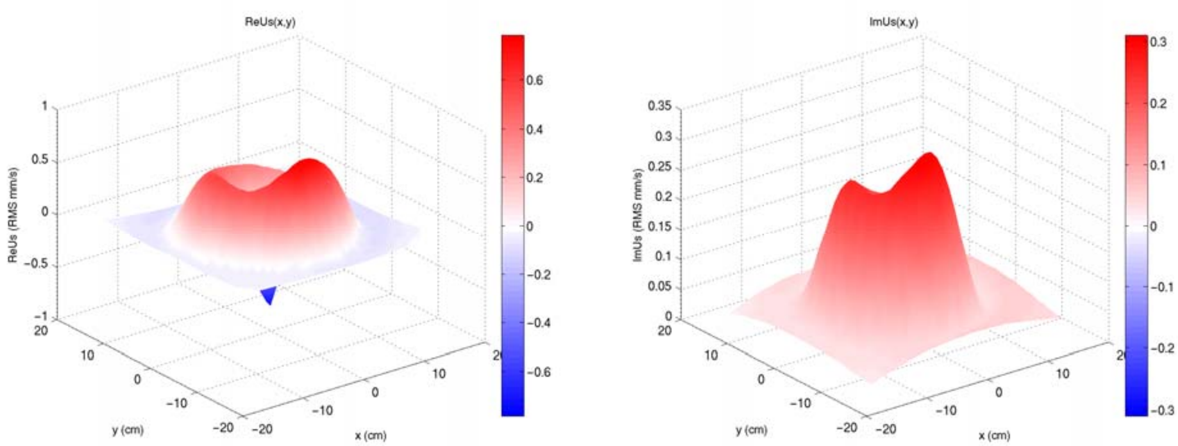

Figure 13: Real and imaginary part of the velocity for a J_{02} of a doumbek.

In Figure 14, Figure 16 and Figure 18 the frequency and phase variations of the measurements are shown for the J_{12}, J_{31} and J_{22} respectively. The resonant frequency of the J_{31} and J_{12} fall 4 Hz to 6 Hz while measuring. It means that these modes are very sensitive to small climate changes and might be narrow resonances that fluctuate a lot.

The velocity field associated with J_{12} is plotted in Figure 15. The resonant frequency is 787 Hz which means that the ratio between this mode and J_{01} is 2.514. The mode is not as clear as the previous ones. This could be due to the fact of frequency and phase changes during measurement. Additionally the grid we are measuring is 32x32 point which means that the resolution is not very high. Therefore a single measurement being wrong could mess up the entire measurements.

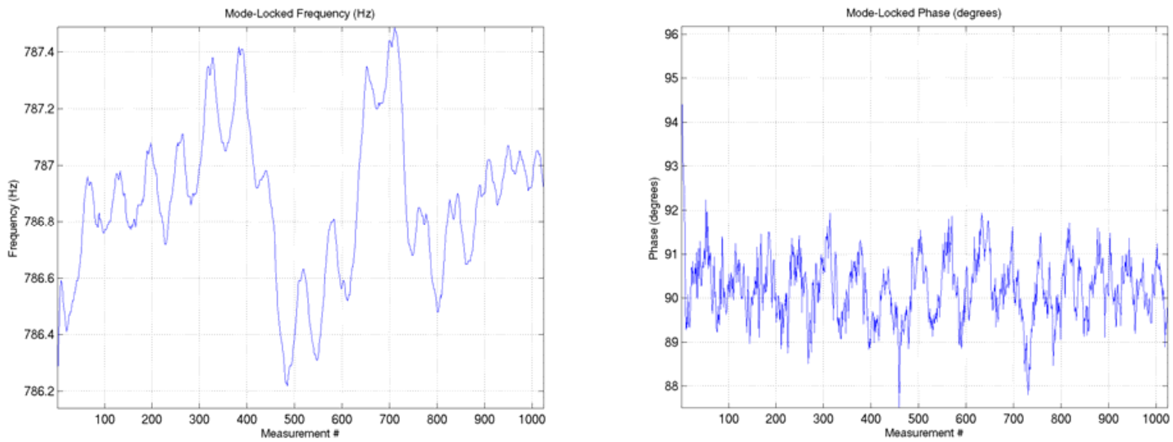

Figure 14: The frequency and phase of J_{12} are shown. The temperature is 17.0° C, the pressure is 748 mmHg and the relative humidity is 24 %. V_{FG} = 3.5 V. The J_{12}-mode has a positive parity.

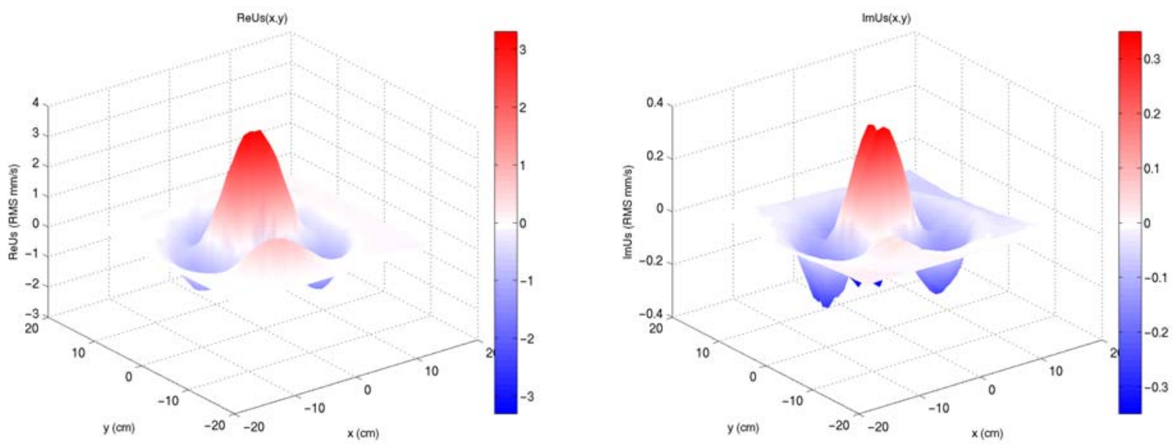

Figure 15: Real and imaginary part of the velocity for a J_{12} of a doumbek.

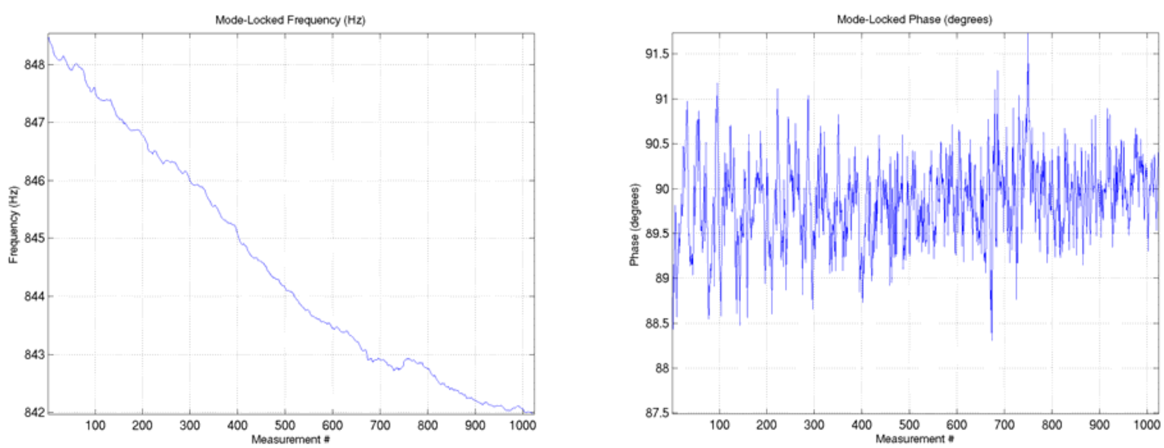

Figure 16: The frequency and phase of J_{31} are shown. The temperature is 18.3° C, the pressure is 750 mmHg and the relative humidity is 21 %. V_{FG} = 3.5 V. The J_{31}-mode has a positive parity.

Figure 17 shows the velocity field of J_{31}. The real and imaginary parts look almost the same. It is hard to determine which eigenmode the drum is vibrating in. On the plots it seems to be 6 large peaks in a circle around the center and the center is a node. This is how a J_{31} looks. The resonant frequency ratio between J_{31} and J_{01} is 2.70.

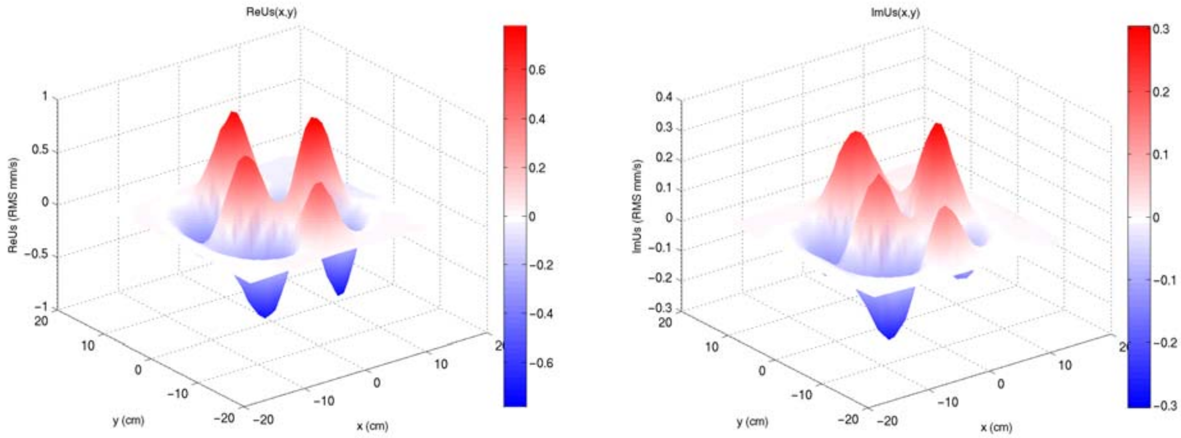

Figure 17: Real and imaginary part of the velocity for a J_{31} of a doumbek.

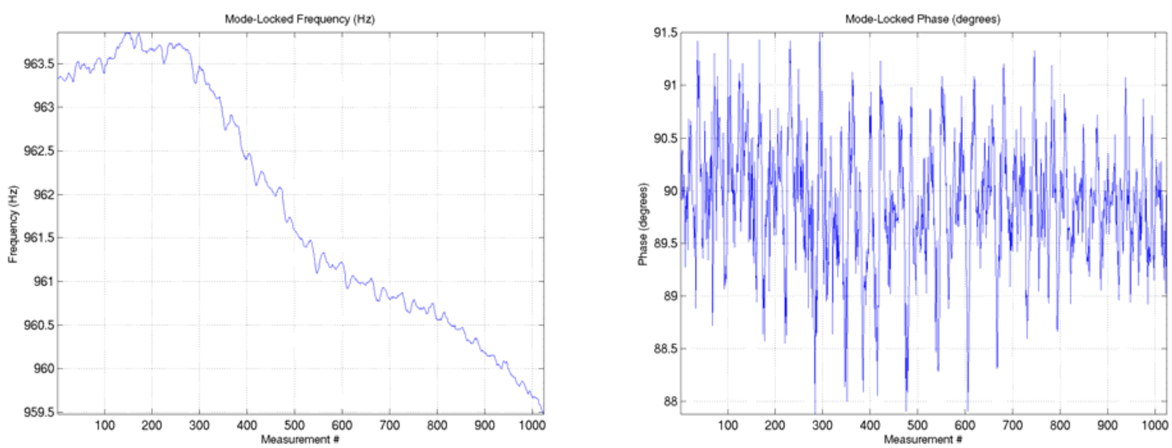

Figure 18: The frequency and phase of J_{22} are shown. The temperature is 17.4° C, the pressure is 749 mmHg and the relative humidity is 23 %. V_{FG} = 2.5 V. The J_{22}-mode has a positive parity.

In Figure 19 the real and imaginary part of the velocity field are shown for a resonances frequency 845 Hz. From the modal vibration of both the real and imaginary part of the velocity field it is hard to determine which Bessel function it is. The imaginary part looks like a J_{02}, which it cannot be because we already have measured this mode and the drum only has one resonance associated with mode. Therefore it looks to be a J_{22} resonance. The ratio between this resonance and the J_{01} is 3.07.

The resonance frequencies investigated is not coming in the same order as the theory suggests that the 6 lowest resonances measured should come in. Further we have not detected J_{21} and J_{41} even thought they should have a lower resonant frequency lower than some of the measured. These resonances could be heavily damped which makes them difficult to excite and both resonances have a node in the center which makes the coupling to the drum hard.

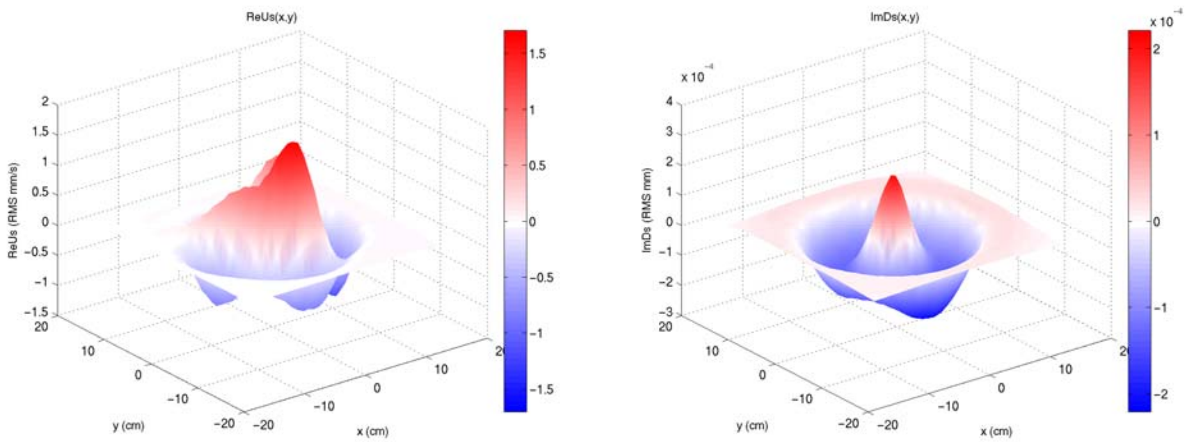

Figure 19: Real and imaginary part of the velocity for a J_{22} of a doumbek.

The measurements show how the different resonances of a doumbek lie and how they differ from chose of an ideal membrane. The reason for the change in resonances could be due to the shape of the drum by having almost a close Helmholzt's resonator below the membrane. This resonator definitely changes the acoustic of the drum. Further by putting a lot of measuring equipments into the cavity also changes the sound and resonances associated with the drum because it changes the air flow inside the drum.

Investigation of Brazilian Drum

Now we can continue by investigating a Brazilian Drum. The drum has a different shape than the doumbek. One would expect the Brazilian Drum to have different resonance frequencies. The drum is a lot bigger than the doumbek which means one has to change the setup so the drum fits under the microphones. The membrane is made out of skin which is thicker than the plastic membrane on the doumbek. The thicker skin and structure to it which influence the sound coming form the drum. Also the long cylinder acts as a resonator increasing the sound heard from the drum.

As one did with the doumbek a frequency scan of the Brazilian Drum is made in order to estimate the resonance frequencies. Figure 20 shows the results of the measurements; the circles denote a resonance and stars denote anti-resonances. The Brazilian Drum has less resonance in the frequency span from 1 Hz to 2 kHz than the doumbek shown in Figure 5, this means that the Brazilian drum will not have as many distinct frequencies associated with its sound. If one have ever played a Brazilian drum one will notice that is has a different sound than other drums, this could be due to the fewer and broader resonances the drum has.

The modal vibration is measured for resonance number 1 at 190 Hz, number 2 at 298 Hz, number 3 at 565 Hz and number 4 at 620 Hz. The frequency of the resonances is approximate values because the frequency changes significantly doing measurement for the resonances as shown later. This could be due to climate changes which might have a bigger effect on the skin than on plastic.

Figure 21 shows the phase and frequency changes during measurement for the J_{01} mode. The real and imaginary part of the velocity field slightly above the membrane is shown in Figure 22. The modal vibration looks very similar as the theoretical prediction of a J_{01} mode.

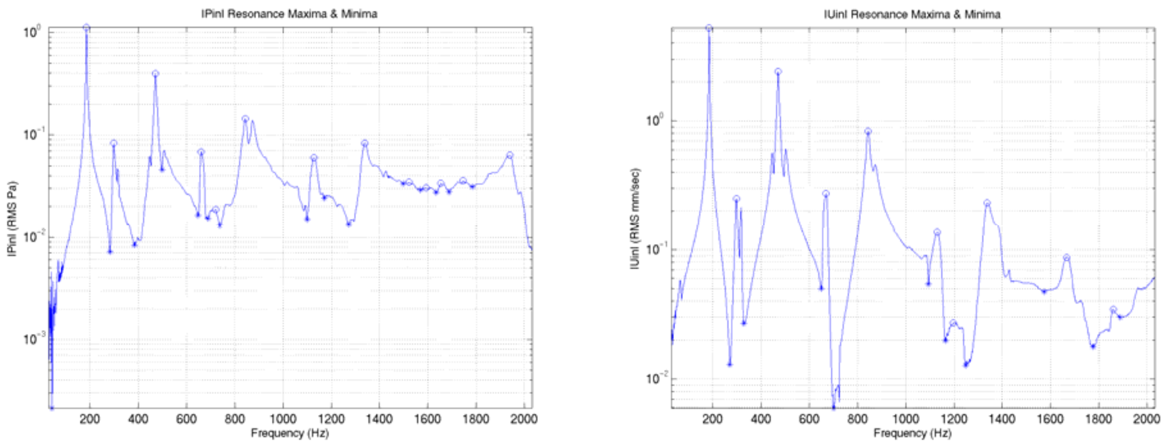

Figure 20: The pressure and the particle velocity of the Brazilian drum are measured. The circles denote resonances and stars denote anti resonances. The measurement is done with 1 Hz step in frequency which gives a high enough resolution to get all the resonance.

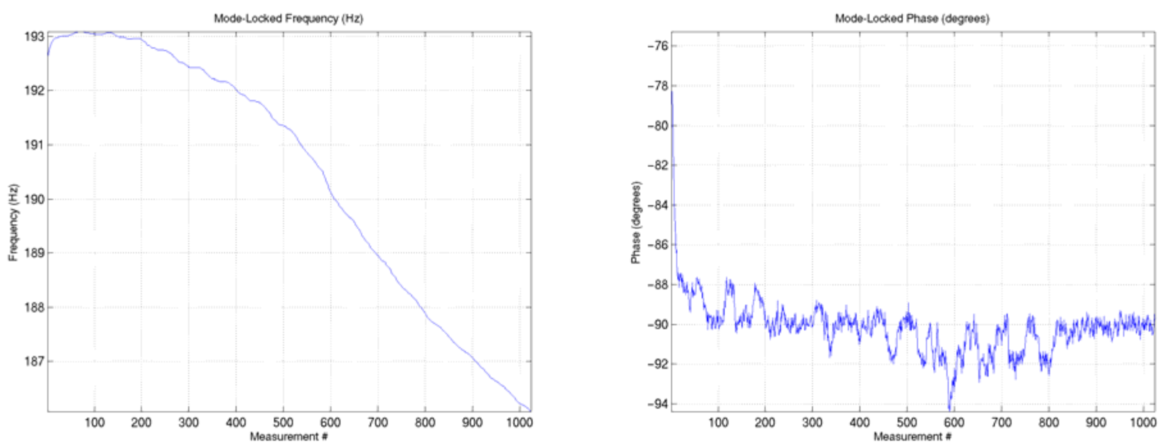

Figure 21: The frequency and phase of J_{01} are shown for the Brazilian drum. The temperature is 26.0° C, the pressure is 743 mmHg and the relative humidity is 41 %. V_{FG} = 5.0 V. At the end of the measurement the temperature is 25.2° C the relative humidity is 46 %. The J_{01}-mode has a negative parity.

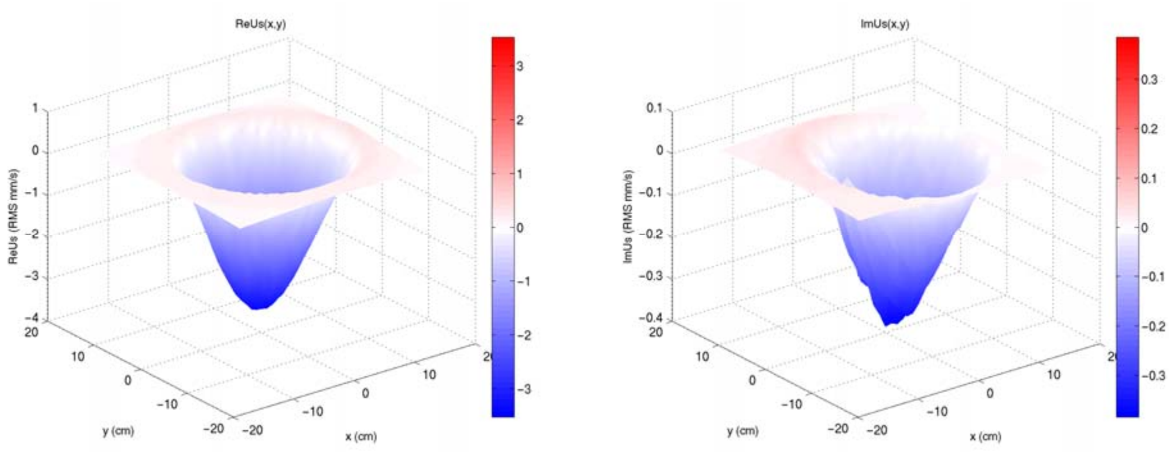

Figure 22: Real and imaginary part of the velocity for a J_{01} of a Brazilian drum.

Figure 23 shows the frequency and phase of the J_{11} mode. During the measurement a sudden change in phase going from 90° out of phase to -25° out of phase. This could mean that something is wrong with the measurements, the same abrupt change is also observed in the resonance frequency that 4 Hz within a couple of measurements, especially when the change is bigger than the change of all the rest of the measurements.

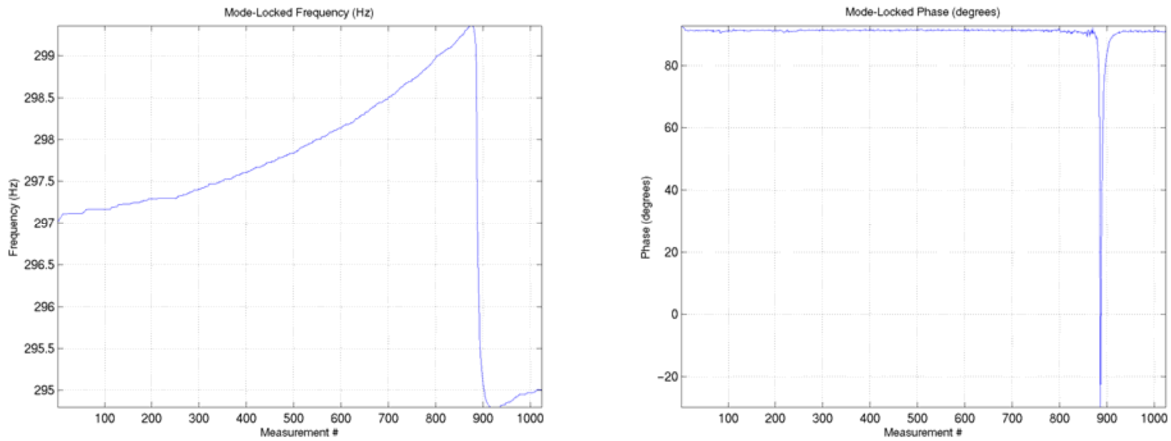

Figure 23: The frequency and phase of J_{11} are shown for the Brazilian drum. Note the sudden change in phase and frequency. This could be the reason Figure 24 does not look very much like a J_{11} mode. The temperature is 23.7° C, the pressure is 745 mmHg and the relative humidity is 49 %. V_{FG} = 10.0 V. The J_{01}-mode has a positive parity.

The eigenmode of J_{11} is shown in Figure 24 as indicated before the mode does not look as it should theoretically. The only indication of a J_{11} mode is the imaginary part of the velocity perpendicular to the surface. So in order to improve the measurement one has to make a new measurement of this mode, but this is however not done due to lack of time. Instead the aim is to focus on other resonant frequencies. Even though how bad the measurement is it was possible to figure out which resonant mode it is. The ratio between this frequency and the J_{01} is 1.57.

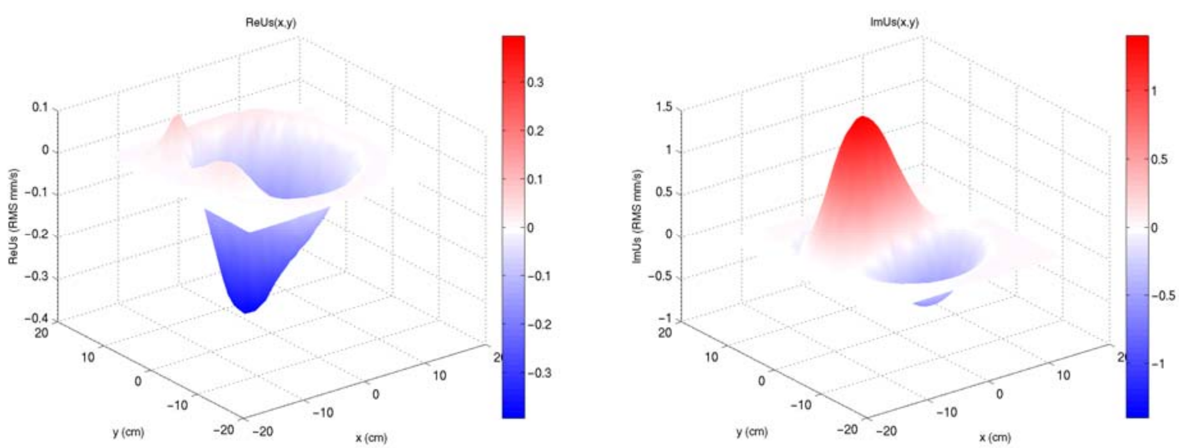

Figure 24: Real and imaginary part of the velocity for a J_{11} of a Brazilian drum.

Continuing with other resonant frequencies Figure 25 shows the frequency and phase changes during the measurement of J_{02}. The frequency increases while measuring but the phase keeps more or less constant.

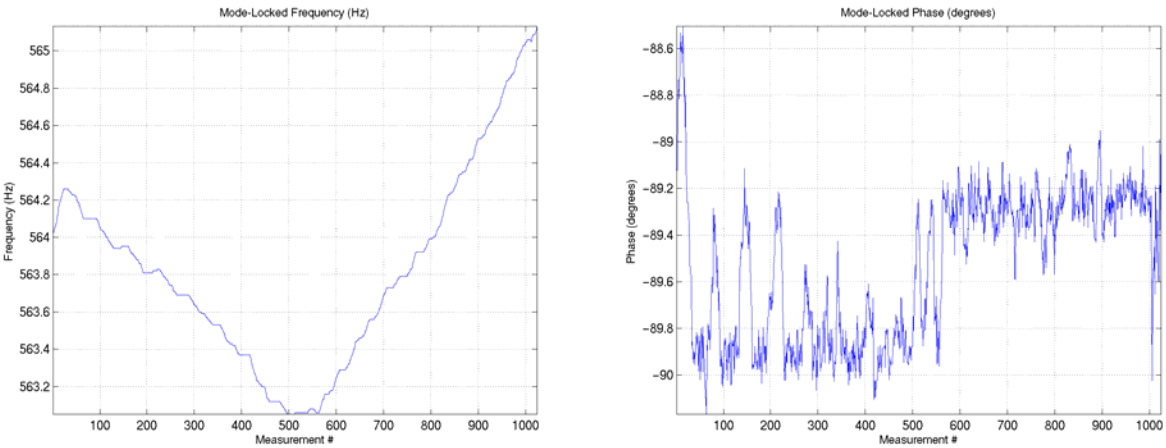

Figure 25: The frequency and phase of J_{02} are shown for the Brazilian drum. The temperature is 22.8° C, the pressure is 740 mmHg and the relative humidity is 33 %. V_{FG} = 5.0 V. The J_{01}-mode has a negative parity.

This also can be seen in the real and imaginary part of velocity. The mode looks a lot prettier than J_{11}. The mode is a J_{02} which is the 4th lowest resonant mode of a membrane the ratio between this mode and the J_{01} is 2.97. The ratio does not differ that much from the ideal drum.

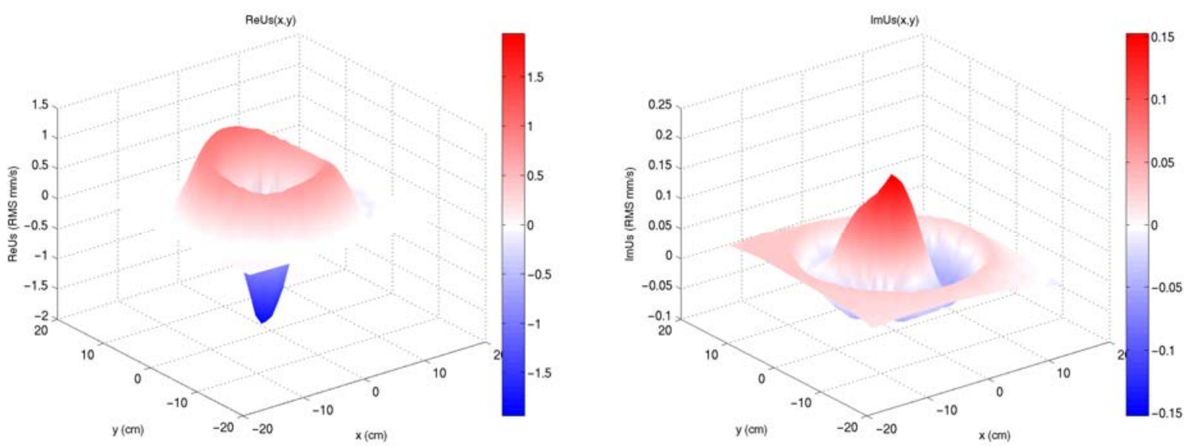

Figure 26: Real and imaginary part of the velocity for a J_{02} of a Brazilian drum.

Figure 27 show the next resonance frequency and phase evolution while measuring. The resonance is a J_{21} which from the theory should have a lower frequency than J_{02}. This is a bit surprising that the Brazilian drum has swapped the order at which the resonance occur in. Another surprising result is how much the frequency increases during the measurement. It almost changes 50 Hz which is a lot especially because one would like the acoustic of the music instrument to be the same. The skin of the drum is probably responsible for this large frequency variation that is much bigger than observed for the doumbek.

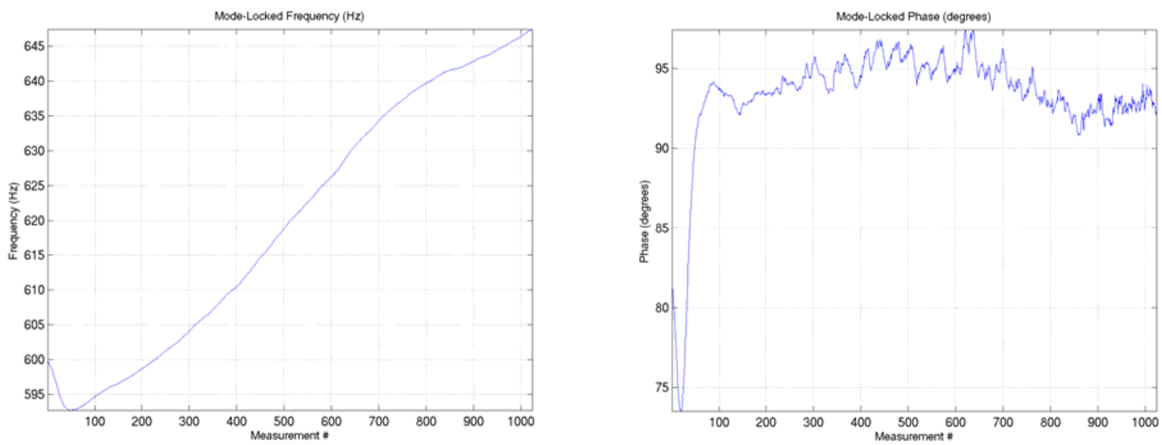

Figure 27: The frequency and phase of J_{21} are shown for the Brazilian drum. The temperature is 23.7° C, the pressure is 745 mmHg and the relative humidity has 49 %. V_{FG} = 10.0 V. The J_{01}-mode is a positive parity.

Figure 28 shows the real and imaginary part of the velocity. One can really see the spatial structure of a J_{21} mode in the real part or the velocity. The ratio between J_{21} and J_{01} is a bit more difficult to determine because of the frequency changes, but it is approximately 3.26.

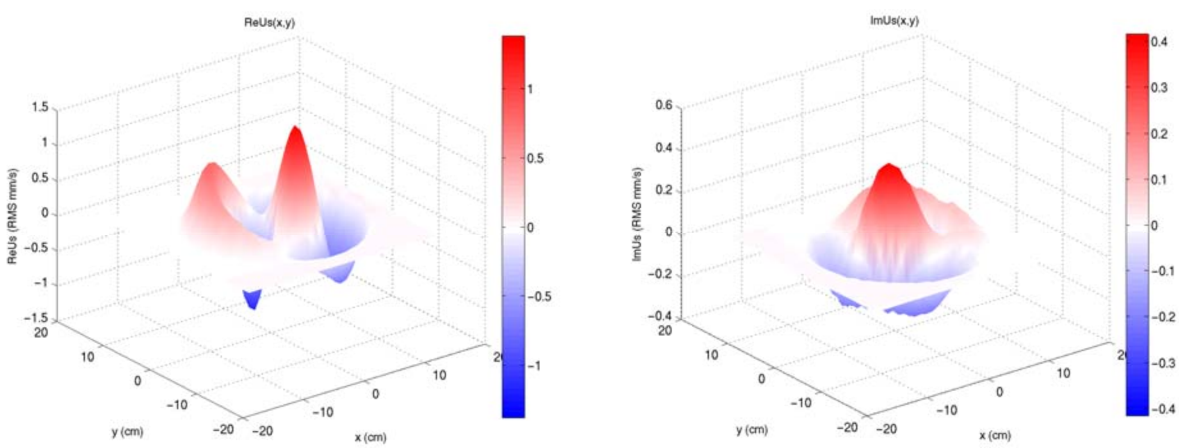

Figure 28: Real and imaginary part of the velocity for a J_{21} eigenmode of a Brazilian drum.

The measurement of two drum and there eigenmodes of different frequencies is found and put into Table 2. The numbers are given relative to the fundamental frequency. One can see that the eigenmodes of the drums do not come in the correct order as the theory predicts. The eigenmodes J_{31} and J_{12} has swapped places for the doumbek as well as J_{21} and J_{02} for for the Brazilian Drum. The reason for swapping places could be due to shape of the drum that it is not only a 2-dimensional structure but a 3-dimensional drum where the membrane couple to the rest of the drum changing the sound compared to a ideal membrane.

Table 2: The ratio between fundamental eigenmodes and higher order modes is shown for an ideal membrane and the two drums investigated in this report. Note that some of the resonances do not come in the correct order. The data from the Tom drum is previous measurements done on this setup.

Membrane	J_{01}	J_{11}	J_{21}	J_{02}	J_{31}	J_{12}	J_{41}	J_{22}
Ideal	1	1.59	2.13	2.29	2.63	2.90	3.14	3.49
Doumbek	1	1.40	-	1.81	2.70	2.51	-	3.07
Brazilian	1	1.57	3.26	2.97	-	-	-	-
Tom drum	1	1.85	2.10	2.29	3.01	3.23	3.44	-

Optical measurement

The measurement has been done using four microphones that makes 1024 measurements on each resonant frequency. This is a long and problematic process because the climate can change as well as the setup seems to be very sensitive to static and other forms of disturbance. Further if some of the 1024 measurements goes wrong it will destroy the measurements which is the case in Figure 23 measuring the J_{11} eigenmode.

A way to improve the measurement is to use optics. Several articles shows by using speckle pattern one can determine the modal vibration of a drum. The method of optical measuring the modal vibration of an object is described and shown in Moore [8] and Demoli [9]. They use a laser source which they split into two one being used as a reference and the other one used to illuminate the object. Capturing the reflected light for the object and mix it on a 50/50 beam splitter on can see the speckle pattern associated with the object. The speckle pattern occurs due to the fact the light hitting different position on the surface has different path length back to the beam splitter. This small delay changes the relative phase of the reflected light and the reference light. The interference pattern is captured by a CCD-camera.

Another optical measurement technique is holographical interferometry. This interferometer will not be discussed but it is similar to speckle pattern interferometer and is explained in Rossing et al.

The method is very sensitive to small changes on the surface of the object due to the high frequency of light. The method also provides a real time measurement so one can see the time evolution of the drum in slow motions. But the biggest disadvantages is that the measurement is a relative measurement meaning that the image does not show the amplitude but the phase different associated by different path lengths. This is however something one can account for by using the physical properties of light.

This method can also be use on other musical instruments than a drum, which does not have a planar surface as long as one have enough power in the laser to illuminate the entire body and recapture the reflected light. Further the setup also allows us to excite the modal vibration of the drum acoustically instead of using two magnets. The magnet mass changes the properties of the drum and thereby the resonances, however an acoustic excitation could be very loud in order to get the membrane vibrating so on can capture the displacement. This is however not very pleasant for us humans. Further the coupling between sound field and membrane could be different than one expect which makes it a bit more unreliable than using magnets. But all in all an optical measurement could improve the data collected and give better and faster results.

Conclusion

In this report one has shown the different frequencies that a drum vibrates in. The real and imaginary part of both the velocity field and the pressure slightly above and below is measured.

Measuring on a drum shows that the shape of a drum definitely changes the acoustical properties of the drum. This is caused by the coupling between the membrane and the body of the drum. However the theoretical prediction fits quite well to the actual measurement and one can clearly see that the eigenmodes are solutions to the equation of motion two dimensional membrane with slightly modifications.
In the theoretical investigation one showed that two membranes with the same circumference and area emit the same frequency spectrum. This is however in an idealized world as the measurement shows. Different membranes, temperature, tension, body and humidity are all factors that could be different and make them sound different.

Acknowledgement
I would like to thank Professor Steven Errede for giving me a physical understanding of acoustics along with helping me with making all the measurements and explaining the setup: I very grateful for the time professor Errede has spend in the lab all semester long to get my measurements finished. I would also like to thank Ben Juday for letting me borrow two of his drum while I was measuring on them.

Knud Palmelund Sørensen

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Appendix

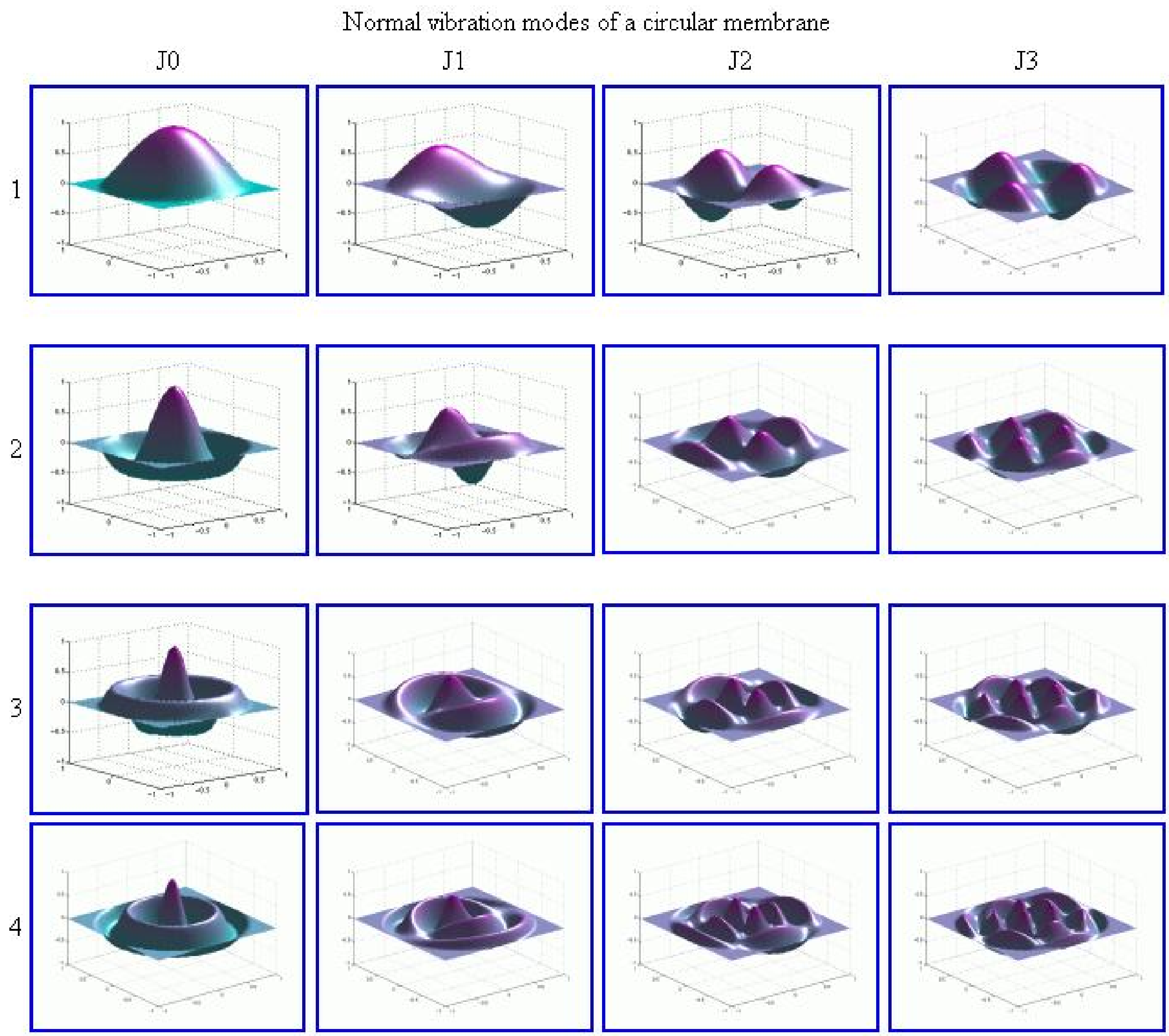

Figure 29: Eigenmodes of an ideal membrane is shown above. The figure was used to determine the resonances. The top row shows J_{01}, J_{11}, J_{21} and J_{31} going from left to right. The next line shows J_{02}, J_{12}, J_{22} and J_{32} and so forth.

Figure 30: The setup is shown in the lab. The drum to the right is the Brazilian drum that we have measured on. The computer is used for collecting data and the four lock-in amplifiers are shown in the middle of the picture two above and two below the

Figure 31: The doumbek is shown under the microphone.

Figure 32: The figure show a Tom drum used in a previous experiment. Through the membrane one can see the two microphones and the electromagnet that is placed below the membrane. The stick coming down from above is holding the two microphones above the membrane.